Abstracts of the III International Meeting on Lorentzian Geometry

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1 Some results on Calabi-Bernstein maximal surfaces in Lorentzian products

Alma Luisa Albujer Brotons. Universidad de Murcia

El teorema de Calabi-Bernstein establece que las únicas superficies maximales completas en el espacio de Lorentz-Minkowski tres-dimensional $\mathbb{L}^3$ son los planos espaciales. En esta charla presentamos nuevos resultados del tipo Calabi-Bernstein para superficies maximales completas en productos lorentzianos del tipo $\mathrm{M} \times \mathbb{R}$ donde M es una superficie riemanniana de curvatura de Gauss no negativa. En particular, probamos que toda superficie completa maximal en $\mathrm{M} \times \mathbb{R}$ ha de ser totalmente geodésica. Además, si M no es llana, las únicas superficies maximales completas en $\mathrm{M} \times \mathbb{R}$ son los "slices" sobre M una superficie necesariamente completa. Por otro lado, en un contexto más general donde no exigimos que la superficie sea completa, hemos obtenido una cota para la norma de la segunda forma fundamental sobre $\mathbb{R}$.

Por último, una formulación equivalente del teorema de Calabi-Bernstein permite enunciarlo afirmando que los únicos grafos enteros maximales sobre $\mathbb{L}^3$ son los grafos afines, es decir, los planos espaciales. De este modo, obtenemos el mismo tipo de resultados Calabi-Bernstein cuando consideramos grafos enteros maximales sobre una variedad M conexa y completa.

2 Lorentzian Sasakian manifolds with constant pointwise $\phi-$sectional curvature

Pablo Alegre Rueda. Universidad de Sevilla

We study Lorentzian almost contact manifolds, (M, ϕ, ξ, η, g), whose curvature tensor satisfies:

$$\begin{aligned} R(X,Y,)Z &= f_1\{g(Y,Z)X - g(X,Z)Y\} + \\ &+ f_2\{g(X,\phi Z)\phi Y - g(Y,\phi Z)\phi X + 2g(X,\phi Y)\phi Z\} \\ &+ f_3\{\eta(X)\eta(Z)Y - \eta(Y)\eta(Z)X + g(X,Z)\eta(Y)\xi - g(Y,Z)\eta(X)\xi\}, \end{aligned}$$

for all $X, Y, Z \in TM$, and certain differentiable functions f_1, f_2, f_3 on M.

This generalizes Lorentzian Sasakian manifolds with constant ϕ-sectional curvature in the same way as generalized complex (resp. Sasakian) space form do with complex (resp. Sasakian) space form.

We present some interesting examples using warped products.

3 Spinors and conformal curvature

Göran Bergqvist. Linköpings Universitet (Sweden)

Spinor methods are very useful when studying the Weyl conformal curvature tensor in four dimensional Lorentzian geometry. We give a quick introduction to 2−spinors and describe some results for which spinor methods provide the only known proofs. New such results include algebraic and differential characterizations of the Bel-Robinson tensor, the superenergy tensor of Weyl tensor.

4 General Relativity from a variational point of view

Anna Maria Candela. Università degli Studi di Bari (Italy)

In these last years variational tools and topological methods have been sistematically used in order to study geodesic properties of some spacetimes

which allow one to describe gravitational fields in General Relativity.
In particular, such an approach allowed one to prove geodesic connectedness in some models of Lorentzian manifolds such as stationary manifolds, Gödel type spacetimes or plane fronted waves.
The aim of the most recent works is removing some technical assumptions introduced in the pioneer papers, so to obtain the existence of geodesics in the most natural setting.

5 Walker metrics and their applications in the study of Osserman manifolds

Eduardo García-Río. Universidade de Santiago de Compostela

The purpose of this lecture is to report on some recent works in the field of pseudo-Riemannian geometry, where Walker metrics are the underlying structure. Usually Walker metrics exhibit many specific features of pseudo-Riemannian geometry and they appear naturally in the study of different problems like hypersurfaces with two-step nilpotent shape operators or in investigating the uniqueness of the metric for a given Levi Civita connection. Walker metrics have also interesting applications in determining some classes of Osserman metrics with non-nilpotent Jacobi operators.

6 Contact numbers of semi-Riemannian submanifolds

Juan Salvador Gómez Casanueva. Universidad de Sevilla

Se define el nmero de contacto de una subvariedad semi-Riemanniana obteniéndose resultados de clasificación según el número de contacto de la subvariedad. Se estudia también la relación entre pseudoisotropía y número de contacto.

7 Spacelike surfaces with nondegenerate second fundamental form

Stefan Haesen. Katholieke Universiteit Leuven (Belgium)

For a spacelike surface with positive definite second fundamental form in any 3−dimensional Lorentzian manifold, a new formula relating its mean and Gauss curvature with the Gauss curvature of the second fundamental form is obtained.
As an application, necessary and sufficient conditions are established in order to get that such a compact spacelike surface is totally umbilical. Moreover, assuming that the shape operator of the surface is a Codazzi tensor, totally umbilicity is also achieved under additional necessary conditions, specially for compact Weingarten spacelike surfaces.

8 A second order variational principle for the Lorentz force equation: conjugacy and bifurcation

Miguel A. Javaloyes. Universidad de Murcia

We give a notion of conjugate instant along a solution of the relativistic Lorentz force equation (LFE); with our definition, magnetic conjugate instants correspond, up to first order infinitesimals, to focalization of solutions having a *fixed* value of the charge-to-mass ratio. We prove a second order variational principle relating every solution of the (LFE) with a canonical lightlike geodesic in a Kaluza-Klein manifold, whose metric is defined using the value of the charge-to-mass ratio. Magnetic conjugate instants correspond to conjugate points along the lightlike geodesic, and therefore they are isolated; based on such correspondence, we prove a bifurcation result for solutions of the (LFE).

9 $5-D$ Lorentzian Geometry Formulation of General Relativity and Beyond

Jaime Keller. Universidad Nacional Autónoma de México

Physics, understood as the science describing nature as a whole in a useful way within the Scientific Method, requires that two observers will find in their experiments similar phenomena and describe them with similar logical structures. This is achieved with a generalized form of the Lorentz Transformations. In our case a flat $5-D$ Lorentz Geometry. For this all matter, including massive objects, and light rays correspond to bundles of null trajectories.
When interactions are included the trajectories of all physical objects remain to be null. A generalization of the Lorentz transformations incude dynamics. The unity of approach and the easiness of the procedures also provides a badly needed didactical procedure to understand theoretical physics.

10 Riemannian metrics as deformations of a flat metric

Josep Llosa. Universitat de Barcelona

It is well known that a (semi-)Riemannian metric in an n-manifold has $n(n-1)/2$ degrees of freedom, that is, it is locally equivalent to specify $n(n-1)/2$ functions. This feature involving some particular choice of local charts, it seems to be coordinate dependent or "non-covariant".

According to this, as it is well known, any two-dimensional metric g is conformally flat, $g = \sigma\eta$, where η is a flat metric and σ is a function (the sole degree of freedom). This statement is stronger than the above Riemann result, indeed, it is *intrinsic* and *covariant* because it only involves tensor quantities: the sole degree of freedom is a scalar function, σ, that only depends on the metric g.

We study the possibility of extending this local property to a higher number of dimensions. That is, we see whether or not there exists a similar intrinsic relation between an arbitrary (semi)Riemannian metric g, the corresponding flat metric η and a covariant set of $n(n-1)/2$ quantities.

We have proved that in the analytic case:

Locally it always exist a 2-form F and a scalar function α such that: (i) *a previously chosen constraint, $\Psi(\alpha, F) = 0$, is met and* (ii) *the (semi)Riemannian deformed metric: $\overline{g}_{\mu\nu} = \alpha g_{\mu\nu} - \epsilon F^2_{\mu\nu}$, with $\epsilon = \pm 1$ and $F^2_{\mu\nu} := F_{\mu\rho} g^{\rho\lambda} F_{\lambda\nu}$, is flat.*

The proof is approached as the solution of a partial differential system. The condition that $\overline{g}$ is equivalent to the vanishing of the Riemann tensor, $\overline{R}_{\mu\nu\alpha\beta} = 0$, that is $n^2(n^2-1)/12$ independent equations. The existence of analytic solutions is then investigated by applying the Cauchy-Kowalevski theorem and the full set of equations is separated in a *reduced partial differential system* and some *constraints* on the Cauchy hypersurface (in a similar way as it is done in the study of existence of solutions to Einstein equations).

11 The lightlike dimensional reduction of the inhomogeneous massless little group into the Galilei group

Ettore Minguzzi. Università degli Studi di Firenze (Italy)

We give an overview of recents results on the transformation properties of shadows under Lorentz transformations that preserve frequency and direction of light. Through a process of lightlike dimensional reduction the relevant inhomogeneous massless little group reduces to the Galilei group of a $1+2$ dimensional quotient spacetime. As a consequence the physics of shadows turns our to be Galilei invariant. We end the talk by giving an example that reveals this symmetry.

12 Massless particles in warped three spaces

Manuel Barros, Magdalena Caballero and Miguel Ortega*. Universidad de Granada

The model governed by an action measuring the total proper acceleration of trajectories provides a nice one to describe the dynamics of massless relativistic particles. In high rigidity cases, metrics with constant curvature, the model is consistent only in spherical three spaces and in three dimensional anti de Sitter backgrounds, according to a Riemannian or a Lorentzian context, respectively. In contrast with flat gravitational fields, the existence of non trivial trajectories are shown in a family of three spaces whose metrics admit a certain degree of symmetry, being such trajectories included in regions with real presence of matter.

13 On the Bonnet problem for compact spacelike surfaces

José Antonio Pastor. Universidad de Murcia

In this talk we present some uniqueness results concerning compact spacelike surfaces in the Lorentzian space forms with the same metric and mean curvature. Since Bonnet, it is well known that a surface cannot be completely determined by the metric and the mean curvature. Therefore, we impose some additional conditions over the boundary of the surface in order to determine the unique compact spacelike surface with prescribed metric and mean curvature function.

14 Multidimensional cosmological models with vanishing Weyl curvature tensor

Miguel Brozos-Vázquez, Eduardo García-Río, Ramón Vázquez-Lorenzo*. Universidade de Santiago de Compostela

The four-dimensional Friedmann-Robertson-Walker (FRW) model provides a useful tool for describing the large scale of the observable part of our present time Universe. However, at Planck distances the spacetime might have some extra dimensions and thus the FRW model is naturally generalized to multidimensional cosmological models (MCMs) with spacetime manifold $M = M_0 \times M_1 \times \cdots \times M_n$ and with decomposed metric $g = g^{(0)} + \sum_{i=1}^{n} e^{2\beta^i(x)} g^{(i)}$, where x are some coordinates on the D_0-dimensional external spacetime M_0 and $g^{(i)}$ are the metrics on the internal spaces M_i $(i = 1, \ldots, n)$.

On the other hand, in locally conformally flat spacetimes the number of unknown functions in the Einstein equations is reduced in contrast with the general case and thus it is worth analyzing the local conformal flatness of MCMs. The purpose of this talk is to report on a local description of locally conformally flat MCMs with a higher dimensional Lorentzian external spacetime. As a consequence we obtain some restrictions on the geometry and the number of the internal spaces. Also we apply these results to analyze some illustrative examples.