Robust Feedforward/Feedback Control for a Class of Nonlinear Systems

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Abstract: In this paper two feedforward/feedback control schemes are derived for a class of nonlinear systems: one is linear and the other is nonlinear. Both schemes are designed to deal with modeling errors within the nonlinear system. We show that the controllers are capable to attenuate input disturbances. In addition, they require neither the measurement nor the structure of the disturbance. The proposed strategies are implemented to an anaerobic digester used for wastewater treatment. Extensive simulations illustrate the performance and robustness.

Keywords: Feedforward/Feedback Control; Robust Control; Wastewater Treatment

1. INTRODUCTION

Several control schemes have been focused on the output regulation problem in the presence of input disturbances. In the most of the cases, the research community has proposed feedback control schemes where the disturbances are partially-known, known or at least, available for measurement. Standard feedback control presents the following disadvantages in the presence of the input disturbances: a) waits until the effect of the disturbance has been felt by the system, before control action is taken; b) suffers degradation of the closed-loop performance for slow systems or with significant dead time; and c) might create instability in the closed-loop performance (Stephanopoulos, 1984; Luyben, 1990). On the other hand, whenever measurements of the disturbances are available, feedforward compensation combined with output feedback, can drastically improve the controller performance. The solution to the associated linear feedforward/output feedback control problem is by now well understood (see e.g. Morari and Zafiriou, 1989; Aström and Wittenmark, 1995). In regard the feedforward/feedback control of nonlinear systems several advances have been obtained as far as the methodological and mathematical framework of differential geometry has been growing (Isidori, 1989; Kravaris and Arkun, 1991). The problem of feedforward compensation of disturbances into the context of exact state-space linearization was addressed by Calved and Arkun, (1988). A general dynamic feedforward static state feedback synthesis problem has been addressed by attenuating the effect of measured disturbances on the system outputs and a well-characterized input-output behavior of the closed-loop system (Daoutidis and Kravaris, 1993). More recently, the feedforward/output feedback control problem for general SISO minimum-phase nonlinear systems was addressed by Daoutidis and Christofides, (1995); where the disturbances are assumed known or available for measurement, which is not always possible.

In the present work, two controllers with structure feedforward/feedback are derived (a linear and nonlinear) for a class of nonlinear systems. Some of the advantages of the proposed controllers are the following: a) they can be easily tuned; b) they do not require a priori knowledge of the uncertain terms related to modeling errors; and c) they need neither the measurement nor the structure of the disturbances. The proposed structure is novel in the sense that reference is the feedforward signal while output is the feedback. The paper is organized as follows. In Section II, we present the class of nonlinear systems and the control
problem is formulated. The development of the controllers with structure feedforward/feedback is presented in Sect. III. In Section IV, an anaerobic digester for wastewater treatment is used as an example to show the performance and robustness of the proposed controller. Finally, some concluding remarks are in Sect. V.

2. THE NONLINEAR SYSTEM AND PROBLEM STATEMENT

In this section, the class of nonlinear systems is presented. The control problem is formulated. Some assumptions are proposed in order to show the contribution of this paper.

2.1 The Class of Nonlinear Systems

The work is focused towards nonlinear systems with the following properties:

(P1) The relative degree $r$ is well-posed and equal to one at a point $x^*(i.e., r = 1)$.

(P2) The system is minimum-phase (i.e., stable zero dynamics).

(P3) Only the output $\eta = h(x)$ is available for feedback, where $h(x)$ is a smooth function.

The nonlinear systems with the above-mentioned properties can be written in the following canonical form (Isidori, 1990):

\[ \dot{z} = \alpha(z, \nu) + \beta(z, \nu) u \]  

\[ \dot{\eta} = \xi(z, \nu) \]  (1.2)

where $L_h(x) = z(z, \nu, L_h(x)) = \xi(z, \nu), u \in \mathbb{E}$ represents the control input, $z \in \mathbb{E}$ stands for the output state and $\mathbb{E}$ denotes the internal dynamics. By assumption the subsystem (1.2) is stable (i.e., minimum-phase).

2.2 Problem Formulation

The objective of this work is to deal with the regulation and servo-control problems for nonlinear systems of the form (1). Let us suppose that the following assumptions hold:

(A1) The system (1) is affected by the disturbance $w$, which is uncertain and unmeasured.

(A2) The functions $\eta(z, \nu)$ and $\xi(z, \nu)$ are affected by the disturbance $w$ present as follows: $z(z, \nu, L_h(x)) = \xi(z, \nu, w), w \in \mathbb{E}$, where $R(z, w) = az + n(z, w)$ with $a \neq 0$ known constant.

From Assumption (A1), it is not possible to deal with the proposed control problem as a classical disturbance rejection or disturbance decoupling problem (Isidori, 1990). Therefore, in the next section two controllers are designed considering the assumptions (A.1) and (A.2). Each controller results from the analysis of the following two study cases:

(C1) Only a nominal value of the perturbation $w$ is known (e.g., if the bounds were known, the nominal value could be computed by the arithmetic mean $\overline{w} = (w_{\max} + w_{\min})/2$). In addition, here we consider the function $(z, w)$ is given by: $(z, w) = (z, \nu) + (\hat{w}, \nu); where (\hat{w})$ is a smooth and known function while $(\nu)$ is an unknown and bounded function.

(C2) The nominal value of the perturbation $w$ is unavailable. Here, we consider that the function $(z, w)$ becomes: $(z, w) = (z, \nu) + (\hat{w}, \nu); where (\hat{w})$ is an unknown and bounded function. $(\nu)$ is a constant different from zero such that the sign $(\nu) = \text{sign}((z, w))$ at $x = 0 \in \mathbb{E}$.

3. CONTROLLER DESIGN

Under the assumptions (A1), (A2) and properties (P1)-(P3), the system (1) can be rewritten as:

\[ \dot{\eta} = \alpha(z, \nu) + \beta(z, \nu) u \]  \hspace{1cm} (2.1)

\[ \dot{\eta} = \xi(z, \nu, w) \]  \hspace{1cm} (2.2)

where $n(z, w)$ represents the nonlinear and uncertain terms associated to the function $R(z, w)$ (i.e. modeling errors).

3.1 The Nonlinear Approach

Let us consider the case (C1), i.e. $(z, w) = (z, \nu) + (\hat{w}, \nu)$. Then, by following the ideas in Femat et al., (1999) one can define a new function as: $O_{NL}/n(z, w) + (\hat{w}, \nu)u$. Thus the system (2) can be written in the following extended state-space representation:

\[ \dot{\eta} = \alpha(z, \nu) + \beta(z, \nu) u \]  \hspace{1cm} (3.1)

\[ \dot{\eta} = \xi(z, \nu, w) \]  \hspace{1cm} (3.2)

where the uncertain terms $n(z, w)$ and $(\hat{w}, \nu)$ have been lumped in the function $O_{NL}$, which is interpreted as an extended state. Moreover, the states of subsystem (3.1) can be estimated from measurements of the output by means of a high gain Luenberger observer of the following form (Teel and Praly, 1995):

\[ \dot{\hat{\eta}} = \alpha(z, \nu) + \beta(z, \nu) u \]  \hspace{1cm} (4)

\[ \dot{\hat{\eta}} = \chi(z, \nu, w) \]  \hspace{1cm} (4)

where the coefficients $g_1$ and $g_2$ can be chosen such that the polynomial $s^2 + g_s + g_1 = 0$ is Hurwitz. In this way, one can guarantee that the estimation error vector $\epsilon(t) = [z \xi O_{NL}(\hat{\eta})] \epsilon \in \mathbb{E} \times t \in [6.4]$. As a consequence, the following control law yields asymptotic global stabilization of the output toward a desired value $z^*$ (for details see Femat et al., 1999):

\[ u = \frac{1}{\beta(z, \nu)} \left[ - \alpha z - \eta_{\text{NL}} + Kp(z - z^*) \right] \]  \hspace{1cm} (5)

where $z^*$ is the reference signal; which can be constant or a smooth time-variant function. $Kp$ is a negative defined constant, and is related with the convergence rate. Thus, the control is composed by the equations
(4) and (5). Note that from the analysis of the first study case, an output feedback nonlinear control with a dynamic uncertainties estimator is obtained.

3.2 Reference-Feedforward/Output-Feedback Control

Now, let us consider the second study case, i.e., \( \{z,w\} = 0 + \{\theta, \nu\} \), where \( 0 \) is a given constant different from zero such that the sign (0) = \( \text{sign} (\{z,w\}) \) at \( 0 \). Thus, by defining \( \Omega = \Omega(z,w) + \{\theta, \nu\}u \). Since \( \Omega(z,w) + \{\theta, \nu\} \) are unknown the augmented function \( \Omega \) lumps the uncertainties. By following the ideas in the last subsection, the control law becomes:

\[
\dot{z} = az + \hat{\eta}_L + \hat{\varphi}u + g_1(z - \hat{z}) \quad (6.1)
\]

\[
\hat{\eta}_L = g_2(z - \hat{z}) \quad (6.2)
\]

\[
u = \frac{1}{\hat{\varphi}} \left[ az - \hat{\eta}_L + Kp(z - z^*) \right] \quad (6.3)
\]

Note that, since \( 0 \neq 0 \) is constant, the system (6) is linear. In order to investigate its properties, the controller (6) can be taken in the Laplace domain, where, without lost of generality it is assumed that \( s(0) = 0 \). Thus, by combining the subsystems (6.1) and (6.2), the following transfer function of the uncertainties estimator is obtained:

\[
\hat{\eta}_L(s) = g_2 \left[ \frac{(s - a)Z(s) + \varphi U(s)}{s^2 + sg_1 + g_2} \right] \quad (7)
\]

Now, let us define \( E(s) = Z(s) + Z^*(s) \) as the control error. Then, transforming Eq. (6.3), by substituting (7), the following transfer function of the controller is obtained:

\[
U(s) = \frac{1}{\hat{\varphi}} \left[ \frac{(Kp - a) + \frac{g_2}{s}}{s} \left( \frac{Kp + g_1}{s + g_1} \right) \right] E(s) + \frac{1}{\hat{\varphi}} \left[ a + \frac{g_2}{s} \left( \frac{g_1}{s + g_1} \right) \right] Z^*(s) \quad (8)
\]

\[
U(s) = K_1(s)E(s) + K_2(s)Z^*(s)
\]

The controller (8) has two parts. The first one is related to the control error. This part has a proportional-integral (PI) structure in conjunction with a correction term of the integral action (Alvarez-Ramirez et al., 1997), i.e., first part has a feedback configuration. In regard to second part, the controller is related with the reference value. This part has the same structure than those of the first part. However, the second part has a feedforward configuration. The block diagram of controller (8) is illustrated in Fig. 1.

4. AN ILLUSTRATIVE EXAMPLE

Here, we present the numerical implementation of the above-developed controllers. To this end, an anaerobic digester for wastewater treatment is used as illustrative example.

4.1 The Dynamical Model Satisfies Assumptions

We consider an anaerobic digester for the treatment of distillery vinasses by Bernard et al., (2001), whose dynamical model is given by:

\[
\begin{align*}
\dot{x}_1 &= \left( \mu_1 - \alpha D \right) x_1 \\
\dot{x}_2 &= \left( \mu_2 - \alpha D \right) x_2 \\
\dot{x}_3 &= \left( x_{3,in} - x_3 \right) D \\
\dot{x}_4 &= \left( x_{4,in} - x_4 \right) D - k_1 x_4 \\
\dot{x}_5 &= \left( x_{5,in} - x_5 \right) D + k_1 x_4 + k_2 x_5 \\
\dot{x}_6 &= \left( x_{6,in} - x_6 \right) D + q c + k_4 x_4 + k_5 x_5
\end{align*}
\]

where the states variables respectively stand for: \( x_1 \), acidogenic bacteria concentration (g/L); \( x_2 \), methanogenic bacteria concentration (g/L); \( x_3 \), total alkalinity (mmol/L); \( x_4 \), chemical oxygen demand (COD,g/L); \( x_5 \), volatile fatty acids (VFA, mmol/L) and \( x_6 \), total inorganic carbon concentration (TIC, mmol/L). State variable \( x_3 \) is given by the sum of VFA\(^s\) and bicarbonate concentrations. The terms \( x_{4,in}, x_{5,in}, x_{6,in} \) represent the influent composition. The dilution rate, \( D \), is defined as the ratio of the influent flow rate \( (Q_{in}) \) over the reactor volume \( (V) \). Parameter \( a \) denotes the process heterogeneity \( (a = 0 \) corresponds to an ideal fixed bed reactor, whereas \( a = 1 \) corresponds to an ideal continuous stirred tank reactor or CSTR). Parameter \( q c \) represents the \( CO_2 \) molar flow rate which can be computed using Henry\(^s\) law (i.e., \( qc = k_{al} CO_2 \times K_{pC} \)), where \( k_{al} \) represents the liquid-gas transfer coefficient, \( K_{pC} \) the Henry\(^s\) constant and \( P_C \) the \( CO_2 \) partial pressure. The model approach the specific grow of acidogenic and methanogenic bacteria by a Monod and Haldane kinetic expressions; which are respectively given:

\[
\begin{align*}
\mu_1 (x_4) &= \mu_{1,\max} \frac{x_4}{x_4 + K_{S1}} \\
\mu_2 (x_5) &= \mu_{2,\max} \frac{x_5}{x_5 + K_{S2} + \left( \frac{x_5}{K_{I2}} \right)^2}
\end{align*}
\]

where \( \mu_{1,\max}, K_{S1}, \mu_{2,\max}, K_{S2}, \) and \( K_{I2} \) are kinetic constants.

When anaerobic digestion is used for wastewater treatment purposes, the control objective is the regulation of the concentration of the effluent organic
matter at any prescribed concentration in the face to fluctuations at the influent composition and environmental conditions. Such an objective is accomplished by manipulating the dilution rate (Dochain and Bastin, 1986). In this way, the control objective for the anaerobic digester (9) can be stated as follows: it is desired to achieve the COD regulation at a given value (i.e., \(x^*_j\)) which represents the desired COD concentration at the effluent composition) against fluctuations at the influent composition and modeling errors by taking \(D\) as a control input.

The following properties are satisfied by the anaerobic digestion model (9) under normal operating conditions (Bernard et al., 2001; Steyer et al., 2002b): (i) \(D\) is bounded for practical operating conditions; that is, \(\mathcal{B} < 4\) and \(D > 0\) (for the anaerobic digester (9) such values correspond to the wash-out condition \(\mathcal{B} = 0.05\) hr\(^{-1}\) and the minimum permissible to avoid emptying the reactor \(D = 0.002\) hr\(^{-1}\)); (ii) The yield coefficients \(k_i\), \(i = 1,2,3,4\), and the kinetic parameters \(K_{x_3}, K_{x_2}\) and \(K_{x_1}\) are positive constants, while the influent composition \(x_{in,j} = 3,4,5,6\) is piecewise non-negative uncertain constants; (iii) The inequality \((x_{in,j} > 0, j = 3,4,5,6\) is satisfied. This means that while the bacterial culture stays active inside of the reactor a part of the influent composition \(x_{in,j}\) will be degraded to produce CO\(_2\), VFA and CH\(_4\), decreasing in consequence the effluent composition \(x_j\). According to previous comments it is also important to remark that only in the wash-out condition \(x_{in} = 0\).

Let us rewrite the anaerobic digestion model (9) in the affine form for single input single output (SISO) systems: \(\hat{0} = f(x) + g(x)u\), where \(x\) is the vector of states, \(x \in \mathbb{E}^n\), \(u = D\) and \(f(x), g(x)\) are smooth vector fields given by

\[
\begin{bmatrix}
\mu_{1}x_1 \\
\mu_{2}x_2 \\
\mu_{3}x_3 \\
k_2\mu_4x_1 - k_3\mu_5x_2 - qe \\
[k_2\mu_4x_1 - k_3\mu_5x_2 - qe]
\end{bmatrix}, \quad g(x) = \begin{bmatrix}
-\alpha x_1 \\
-\alpha x_2 \\
(x_{in,j} - x_3) \\
(x_{in} - x_4) \\
(x_{in} - x_5) \\
(x_{in} - x_6)
\end{bmatrix}
\]

The system output is given by the COD concentration, i.e., \(y = x_j\).

Thus, by computing the Lie derivative of the system output \(h(x)\) along the vectors fields \(f(x)\) and \(g(x)\), it is obtained that \(L_fh(x) = (x_{in,j}! x_j)\). Since for normal operating conditions \((x_{in,j} > 0)\), it is easy to shown that system (9) has a relative degree \(r = 1\) which is well-defined out of the wash-out condition. In this way, system (9) can be written in the canonical form (1) as follows:

\[
\dot{z} = (x_{in,j} - z)D - k_1\mu_4V_5(x_{in,j} - z)^{\alpha - 1}
\]

\[
\dot{z} = \frac{V_2V_5}{V_4}(x_{in,j} - z)^{\alpha}[k_2\mu_4V_4 - k_3\mu_5]
\]

\[
\dot{z} = \frac{V_2V_5}{V_4}(x_{in,j} - z)^{\alpha}[k_2\mu_4V_4 - k_3\mu_5 - qe]/V_5
\]

\[\dot{z} = V_4(\mu_4 - \mu_5)
\]

\[
V_5 = \mu_iV_4[1 - \alpha k_iV_4(x_{in,j} - z)^{\alpha - 1}]
\]

It has been proved that this system (11.2) is globally stable under normal operation conditions (Méndez-Acosta et al., 2002). In addition, the system output \(y = x_j\) is available for feedback (Steyer et al., 2002a). Therefore, the dynamical model of the anaerobic digestion (9) satisfies the properties (P1), (P2) and (P3). Hence, system (11) can be rewritten in the form (2) as follows:

\[
\dot{\hat{z}} = k_1\mu_4V_5(x_{in,j} - z)^{\alpha} + (x_{in,j} - z)D
\]

\[
\hat{\eta}_{NL} = \hat{\eta}_{NL}(z, \eta_{NL}, \nu)
\]

\[
\dot{\nu} = \dot{\gamma}(z, \nu, w)
\]

where \(a = 0, \eta_{NL}(z, w) = I_{k_1} < (x_{in,j} ! x_j)^{\alpha} \), \((z, w) = (x_{in,j} l D, w = x_{in}! D)\) is the control input.

4.2 Both Controllers in the Anaerobic Digester

For case (C1), we can assume that a nominal value of the perturbation \(x_{in}\) is known (e.g., \(x_{in} = (x_{in,max} + x_{in,min}) / 2\)). It is important to remark that this assumption is easy to satisfy (e.g., for the vinasses feeding into the reactor used to validate model (9) \(x_{in} \in [16.0 \text{ g/L} ; \text{Steyer et al., 2002}].\). Therefore, system (12) can be written in the form (3) as follows:

\[
\dot{\hat{z}} = \hat{\eta}_{NL}(z, \eta_{NL}, \nu)
\]

\[
\dot{\hat{\nu}} = \dot{\gamma}(z, \nu, w)
\]

where \(\eta_{NL}(z, w) = I_{k_1} < (x_{in,j} ! z)^{\alpha} \), \(k_1 < (x_{in,j} ! z)^{\alpha} \). Then, the following output feedback nonlinear control with a dynamic uncertainties estimator is obtained:

\[
\dot{\hat{z}} = \hat{\eta}_{NL}(z, \eta_{NL}, \nu)
\]

\[
\dot{\hat{\eta}}_{NL} = g_2(z - \hat{z})
\]

\[
u = \frac{1}{(x_{in,j} - z)}[\hat{\eta}_{NL} + Kp(z - x_{in}! z)]
\]

Such a control have been successfully tested in a 1m\(^3\) pilot scale fully instrumented anaerobic digester. The controller (14) showed certain margin of robustness against unmodeled dynamics, uncertainties in the system kinetics, load perturbations and noisy measurements (Méndez-Acosta et al., 2002). Here, we study its feedforward/feedback properties.

Now, let us consider the case (C2) in where \((z, w) = (x_{in,j} l D, w = x_{in}! z)\). By defining \(\eta_{NL}(z, w) = I_{k_1} < (x_{in,j} ! z)^{\alpha} \), the following controller of the form (8) is
obtained:

\[
U(s) = \frac{1}{s} \left[ Kp + \frac{g_2}{s} - \frac{(Kp + g_1)g_2}{s(s + g_1)} \right] E(s) + \frac{1}{s} \left[ \frac{g_2}{s} - \frac{g_2g_2}{s(s + g_1)} \right] X_4^*(s)
\]  

(15)

where \( E(s) = Z(s) \) and without lost of generality it is assumed that \( \beta = 16.0 \text{ g/L} \). Note that controller (15) is a simplification of controller (14) were the feedforward/feedback structure is clear. Also, note that neither controller (14) nor (15) have information about the kinetic uncertain terms. Therefore, if controllers (14) and (15) are able to regulate the effluent organic matter concentration at any prescribed value, then both controllers are robust against modeling errors.

4.3 Numerical Simulations

The performance and robustness of the control laws (14) and (15) are evaluated via numerical simulations. Four different set-point changes were made on the effluent COD concentration. The model parameters used in the simulations are the following: \( k_1 = 42.14, K_H = 16.0, k_2 = 116.5, \mu = 0.5, k_3 = 268.0, t_{\text{max}} = 0.05, k_4 = 50.6, k_{\text{max}} = 0.031, k_5 = 343.6, K_S = 7.1, k_6 = 435.0, K_2 = 9.28, k_{\text{max}} = 0.825, K_{\text{max}} = 16.0 \) and \( P_T = 1.0434 \). The initial conditions used are given by: \( x_1(0) = 0.1358, x_2(0) = 0.4829, x_3(0) = 76.1, x_4(0) = 0.9890, x_5(0) = 8.559 \) and \( x_6(0) = 67.186 \). Such parameter values have been identified to the pilot scale plant where controller (14) was implanted (Bernard, et al., 2001). The control parameters used along the simulation were the same for both control laws and these are given by: \( g_1 = 1.5, g_2 = 0.5 \) and \( Kp = 2.0 \).

In order to consider real operation conditions, the COD concentration fed to the digester was randomly varied around a mean value as is depicted in Figure 2. In addtion, noisy measurements were simulated by taking an uniformly random noise (±0.1 g/L) was added to the measured variable \( x_4 \).

Figure 2. Concentration of the organic matter entering to the digester i.e., \( x_{4_{\text{in}}} \)

It is important to remark that the saturation of the control laws is a particular problem due to the operation conditions of the anaerobic digester taken as example. On the other hand, Figure 3b shows that the energy required for both control laws is quite low once the set-point value is reached. In other words, the control input \( D \) is saturated just after the set-point value is changed. In a real process this behavior is quite desired since it guarantees a safety operation conditions.

5. CONCLUSIONS

In this paper a linear and nonlinear control laws were derived from the geometric analysis of a class of nonlinear systems. The controllers display acceptable performance to deal with the servo-control and
regulation problem in the face to modeling errors and disturbance inputs. In addition, they do require neither the measurement nor the structure of the disturbances. The controllers show a certain margin of robustness against noise in the measurement. Moreover, both controllers are easy to tuning and to implemented. As was pointed out, the control laws present a saturation problem due to the operation conditions of the anaerobic digester. This problem will be addressed in future works.

REFERENCES


