Anti-windup Feedforward/Feedback Control for a Class of Nonlinear Systems


* Depto. Ingeniería Química, Universidad de Guadalajara, 44860, Guadalajara-Jalisco, Mexico.
** Fac. de Ciencias, Universidad Autónoma de San Luis Potosí, 78290, San Luis Potosí-S.L.P., Mexico.

Abstract: In this paper an anti-windup scheme is proposed for two robust feedforward/feedback control laws: one linear and the other nonlinear. Since the control laws present an observer-based structure, the anti-windup scheme results in a natural manner by taking into account the actuator constraints. It is shown that the anti-windup scheme improves the performance of the control laws. The resulting strategies are implemented in an anaerobic digester for wastewater treatment. Performance is illustrated by numerical simulations.

Keywords: Anti-windup; Robust Feedforward/Feedback Control; Wastewater Treatment

1. INTRODUCTION

All practical control systems are subject to constraints on the manipulated inputs which are imposed due to the limited capacity of the actuators (valves, pumps, etc.). Particularly, if the control input constraints are not taking into account in the controller design, it may lead to significant deterioration of the control performance and even cause closed-loop instability. Such a performance-stability degradation is due to the so called windup phenomenon. This phenomenon is typically exhibited by dynamic controllers with slow or unstable modes (Doyle et al., 1987).

Recently, numerous anti-windup schemes for a class of nonlinear systems have been proposed. It has been shown that most of them present a “observer-based” structure (Walgama and Sternby, 1990; Kothare et al., 1994). Such a structure can be implemented by constructing the difference between the computed value of the manipulated input and the actual input entering to the process, and feeding back this signal through a static gain to the controller dynamics (Åström and Wittenmark, 1984). On the other hand, the input-output (I/O) linearizing control combined with the optimization-based strategies has been applied in anti-windup strategies for nonlinear systems. A linear anti-windup scheme that preserve the feedback linearization of nonlinear systems subject to constraints was proposed by Doyle, (1999). Kurtz and Henson, (1997) combined an I/O linearizing controller with a linear model predictive controller to regulate constrained nonlinear systems. An alternative anti-windup scheme for nonlinear systems was derived from a finite-horizon optimization (Valluri et al., 1998). Also, an extension of the nonlinear internal model control to systems with constrained inputs and measured disturbances has been proposed (Kendi and Doyle, 1998). Kothare et al., (1994) proposed a novel anti-windup scheme to suppress the gap between the saturated actuator and the computed control input. This scheme comprises an integral-type compensation which replaces the conventional optimization algorithm. A method to design the controller gains and a modified nonlinear observer-based structure to attenuate the effect of windup has been proposed by Kapoor and Daoutidis, (1999). More recently, a modified I/O linearizing feedback control was derived by defining a specific tracking error form and introducing a flexible tuning parameter, which is directly obtained by solving an algebraic calculation (Wu, 2002).

In this paper, an anti-windup scheme is proposed for two feedforward/feedback control laws derived for a class of nonlinear systems. The obtained anti-windup scheme has a natural observer-based structure, which

* Corresponding author. E-mail: rfemat@ipicyt.edu.mx
results from feeding back the measure of the constrained input to the controller. As a consequence, the tuning of the anti-windup scheme is simple. In addition, such a scheme shows an improvement in the performance of the control laws. The rest of the paper is organized as follows. Section II contains the class of nonlinear systems and the problem formulation. The development of the anti-windup scheme is presented in Sect. III. In Section IV, an anaerobic digester for wastewater treatment is used to show the performance and robustness of the proposed scheme. Finally, some conclusions are pointed out in Section V.

2. A CLASS OF NONLINEAR SYSTEMS AND PROBLEM FORMULATION

2.1 The Class of Nonlinear System

The present work is focused towards nonlinear systems with the following properties:

(P1) The relative degree \( r \) is well-posed and equal to one at a point \( x_0 \in \mathbb{E}^n \) (i.e., \( r = 1 \)).

(P2) The nonlinear system is minimum-phase (i.e., stable zero dynamics).

(P3) Only the output \( y = h(x) \) is available for feedback, where \( h(x) \) is a smooth function.

Then, nonlinear systems with the above-mentioned properties can be written in the following canonical form (Isidori and Byrnes, 1990):

\[
\begin{align*}
\dot{z} &= \alpha(z, \nu) + \beta(z, \nu) u, \\
\dot{\nu} &= \xi(z, \nu)
\end{align*}
\]  

(1.1) (1.2)

where \( L \alpha (x) = \nu (x, \phi) \) and \( L \beta (x) = \phi (x, \phi) u \), with given constant \( \phi \neq 0 \). \( h(x) \) represents the control input, \( z \in \mathbb{E}^n \) stands for the output state and \( x \in \mathbb{E}^{n+1} \) denotes the internal dynamics. In addition, subsystem (1.2) is stable when \( y = 0 \) (Property P3).

2.2 Problem Statement

The objective of this paper is to address the regulation problem of the nonlinear systems in the form (1) when the control input \( u \) is constrained. To this end, let us suppose that the following assumptions hold:

(A1) The system (1) is disturbed by a disturbance \( w \) which is uncertain and unmeasured.

(A2) Under disturbances \( w \), the Lie derivatives of subsystem (1.1), affected by the disturbance \( w \), has the form:

\[
L \alpha (x, w) = \mathcal{R}(z, w) \text{ and } L \beta (x, w) = \mathcal{S}(z, w) u
\]

(2.1) (2.2)

where \( \mathcal{R}(z, w) = a z + \phi (z, w) \) with a given constant \( a \neq 0 \).

(A3) The control input \( u \) is subject to constraints. That is, \( u_{sat} = \text{sat}(u) \) and its mathematical representation is given by:

\[
\text{sat}(u) = \begin{cases} 
  u_{\max}, & u \geq u_{\max} \\
  u, & u_{\min} < u < u_{\max} \\
  u_{\min}, & u \leq u_{\min}
\end{cases}
\]

(2)

where the bounds \( u_{\min} \) and \( u_{\max} \) are known.

In the next section two controllers are designed under the above-mentioned assumptions. Each controller respectively results from the analysis of the following study cases:

(C1) Only a nominal value of the perturbation \( w \) is known (i.e., \( \psi = (w_{\max} + w_{\min}) / 2 \)). In this case, the function \( \{g(z, \psi) = (g(z, \psi) + (G z, w) \}

(3.1) (3.2)

where \( (g(z, \psi) \) is known and \( (G z, w) \) is unknown and bounded.

(C2) The function \( \{g(z, \psi) \) is given by:

\[
\psi = \xi(z, \nu, w)
\]

(3.3) (3.4)

where \( (G z, w) \) is unknown and bounded. \( \xi \) is a constant different from zero such that the sign \( (\xi(z, \nu, w) \) at \( x / 0 \) is.

3. ANTI-WINDUP SCHEME

Under assumptions (A1)-(A3) the system (1) with the properties (P1)-(P3) can be written as follows:

\[
\begin{align*}
\dot{z} &= az + \phi (z, w) + \gamma (z, w) \text{sat}(u) \\
\dot{\nu} &= \xi(z, \nu, w)
\end{align*}
\]

(3.1) (3.2)

where \( a \neq 0 \) and \( \phi (z, w) \) represents the nonlinear uncertain terms, which are related to the function \( \mathcal{R}(z, w) \).

It should be noted that the block diagram in Fig. 1 does not include the actuator dynamics. Such a dynamic can affect the closed-loop performance in a physical implementation. Fortunately, the anti-windup performance can be tested by standard SIMULINK package including actuator dynamics (Bohn and Atherton, 1995). Now, in regard to the stability analysis, it had been reported that single loop systems with anti-reset windup schemes are robust compared to its nominal input-output response (Glattfelder and Schaufelberger, 1983). Such results were reported for linear plants; however, an extension to nonlinear systems can be obtained by considering the cross section defined by nonlinear constraints and control surfaces (Alonso and Banga, 1998).

3.1 An Anti-windup Scheme For A Nonlinear Control

In a previous work, Méndez-Acosta et al., (2003), by departing from the case (C1) proposed an output feedback nonlinear control with an uncertainties dynamic estimator for nonlinear systems of the form (3) when the control input is not constrained. Such a controller is given as follows:

\[
\begin{align*}
\dot{z} &= az + \hat{\gamma}_L + \bar{\gamma}(z, \bar{w}) u + g_1(z - \hat{z}) \\
\dot{\hat{z}} &= g_2(z - \hat{z}) \\
& u = \frac{1}{\bar{\gamma}(z, \bar{w})} [ - az - \hat{\gamma}_L + K p(z - z^*) ]
\end{align*}
\]

(4.1) (4.2)

The control law (4) is composed by a proportional-integral (PI) structure given by: a) the difference between the output and the desired value with proportional constant \( K p \); and b) the integral action induced by the term \( \frac{\dot{\gamma}_L}{\bar{\gamma}(z, \bar{w})} \). Then, under assumption (A3), the control law (4) can reach
saturation values due to a sudden change in the inputs. In such a situation, the feedback is broken and the plant behaves as an open-loop with a constant input, allowing a possible degradation of the controller performance. Moreover, due to the integral action, the control saturation can induce undesirable effects such as a large overshoot and settling time (Kothare et al., 1994). This phenomenon is known as reset windup. From the implementation point of view, reset windup appears due to the integral action since the controller does not have knowledge of the control input saturation, so it continues integrating even if the feedback is broken.

On the other hand, note that for the control law (4), there exists an internal feedback between the computed control input (4.2) and the observer used as dynamic estimator (4.1). This means that the observer performance is affected by the computed control input. Then, the problem of reset windup in the controller (4) can be treated in a natural way by feeding back the control input under the constraint described in (2) to the observer. In this way, the controller (4) can be rewritten in the following form:

\[
\dot{z} = az + \hat{\eta}_L + \bar{\gamma}(z, \bar{w})u_{sat} + g_1(z - \dot{z})
\]

\[
\hat{\eta}_L = g_2(z - \dot{z})
\]

\[
u = \frac{1}{\bar{\gamma}} [-az - \hat{\eta}_L + Kp(z - z^*)]
\]

where \(u_{sat}\) is given by Eq. (2). Thus, the controller (5) presents an observer-based anti-reset windup structure (see Figure 1).

3.2 Reference-Feedforward/Output-Feedback Control With Anti-windup

Now, for the study case (C2), Méndez-Acosta et al., (2003) proposed a reference-feedforward/output feedback structure for nonlinear systems of the form (3) without considering the constraints in the control input \(u\). Such controller can be written in the state-space as follows:

\[
\dot{z} = az + \hat{\eta}_L + \bar{\gamma}u + g_1(z - \dot{z})
\]

\[
\hat{\eta}_L = g_2(z - \dot{z})
\]

\[
u = \frac{1}{\bar{\gamma}} [-az - \hat{\eta}_L + Kp(z - z^*)]
\]

Then, following the same ideas reported in the last subsection, it is easy to show that the control law (6) can be rewritten as follows:

\[
\dot{z} = az + \hat{\eta}_L + \bar{\gamma}u_{sat} + g_1(z - \dot{z})
\]

\[
\hat{\eta}_L = g_2(z - \dot{z})
\]

\[
u = \frac{1}{\bar{\gamma}} [-az - \hat{\eta}_L + Kp(z - z^*)]
\]

where, once again, \(u_{sat}\) is given by Eq. (2). The control law (7) is conformed by a linear part given by

\[
K(s) = \begin{bmatrix}
-g_1 & 1 & a + g_1 & 0 & 1/ar{\gamma} \\
-g_2 & 0 & g_2 & 0 & 0 \\
0 & -1/\bar{\gamma} & (Kp - a)/\bar{\gamma} & -Kp/\bar{\gamma} & 0
\end{bmatrix}
\]

which is connected with the saturation function (2). Thus, the linear controller \(K(s)\) is a transfer matrix with three inputs \(u, z, u_{sat}\) and one output respectively given by: \(z^*, z, u_{sat}\) and \(u\). The coefficients \(g_1\) and \(g_2\) are chosen such that the polynomial \(\dot{s} + g_1s + g_2 = 0\) be Hurwitz. The block diagram of the controllers (5) and (7) is depicted in the Figure 1.

4. ILLUSTRATIVE EXAMPLE

In order to implement the controllers developed in the last section, an anaerobic digester for wastewater treatment is used.

4.1 The Model Satisfies the Assumptions and Properties

Consider the control of an anaerobic digester used in the treatment of distillery vinasses (Bernard et al., 2001):

\[
\begin{align*}
\dot{x}_1 & = (\mu_1 - aD)x_1 \\
\dot{x}_2 & = (\mu_2 - aD)x_2 \\
\dot{x}_3 & = (x_{x, in} - x_3)D - k_1x_1 \\
\dot{x}_4 & = (x_{x, in} - x_4)D - k_2x_1 \\
\dot{x}_5 & = (x_{x, in} - x_5)D + k_3x_1 - k_4x_2 \\
\dot{x}_6 & = (x_{x, in} - x_6)D - qc + k_4x_1 + k_5x_2
\end{align*}
\]

where the states variables respectively stand for: \(x_1\), acidiogenic bacteria concentration (g/L); \(x_2\), methanogenic bacteria concentration (g/L); \(x_3\), total alkalinity (mmol/L); \(x_4\), chemical oxygen demand (COD, g/L); \(x_5\), volatile fatty acids (VFA, mmol/L) and \(x_6\), total inorganic carbon concentration (TIC, mmol/L). State variable \(x_3\) is given by the sum of VFA and bicarbonate concentrations. The terms \(x_{x, in}\) represent the influent composition. The dilution rate, \(D\), is defined as the ratio of the influent flow rate \(Q_\text{in}\) over the reactor volume \(V\). Parameter \(\mu\) denotes process heterogeneity \((\mu = 0\) corresponds to an ideal fixed bed reactor, whereas \(\mu = 1\) corresponds to an ideal continuous stirred tank reactor or CSTR). Parameter \(qc\) represents the CO₂ molar flow rate which can be computed using Henry’s law (i.e., \(qc = k_7a(\text{CO}_2)(K_pP_\text{CO}_2)\)), where \(k_7a\) represents the liquid-gas transfer coefficient, \(K_p\) the
Henry’s constant and \( P_C \), the CO2 partial pressure). The model approach the specific grow of acidoenic and methanogenic bacteria by a Monod and Haldane kinetic expressions; which are respectively given by the following functions:

\[
\begin{align*}
\mu_1(x_t) &= \mu_{1,\text{max}} \frac{x_4}{x_4 + K_{S1}} \\
\mu_2(x_t) &= \mu_{2,\text{max}} \frac{x_5}{x_5 + K_{S2} + \left( \frac{x_5}{K_{I2}} \right)^2}
\end{align*}
\]

where \( : 1,\text{max}, K_{S1}, : 2,\text{max}, K_{S2}, \) and \( K_{I2} \) are kinetic constants.

When anaerobic digestion is used for wastewater treatment purposes, the control objective is the regulation of the effluent organic matter at a prescribed concentration in the face of fluctuations at the influent composition and modeling errors by taking \( D \) as a control input. Under normal operation conditions, the control input \( u = D \) is constrained by:

\[
sat(D) = \begin{cases} 
D_{\text{max}}, & D \geq D_{\text{max}} \\
D, & D_{\text{min}} < D < D_{\text{max}} \\
D_{\text{min}}, & D \leq D_{\text{min}} 
\end{cases}
\]

where \( D_{\text{sat}} = sat(D), D_{\text{max}} = 0.05 \, \text{hr}^{-1} \) and \( D_{\text{min}} = 0.002 \, \text{hr}^{-1} \) respectively correspond to the value of wash-out condition and the minimum permissible value which may be used to avoid emptying the digester (Steyer et al., 2002a). Note that the saturation function is not symmetric; as a consequence, it is expected that such property has a strong influence in the controller performance.

Now, let us rewrite the anaerobic digestion model (8) in the affine form for SISO nonlinear systems: \( \dot{x} = f(x) + g(x)u \), where \( x \) is the vector of states, \( x \in \mathbb{R}^n, u = D \) and \( f(x), g(x) \) are smooth vector fields given by

\[
\begin{align*}
f(x) &= \begin{bmatrix} 
\mu_1 x_1 \\
\mu_2 x_2 \\
-k_1 \mu_1 x_1 \\
k_2 \mu_1 x_1 - k_3 \mu_2 x_2 \\
k_4 \mu_1 x_1 - k_5 \mu_2 x_2 - qc
\end{bmatrix}, \\
g(x) &= \begin{bmatrix} 
-x \bar{x}_1 \\
-x \bar{x}_2 \\
(x_{x,\text{in}} - x_3) \\
(x_{x,\text{in}} - x_4) \\
(x_{x,\text{in}} - x_5)
\end{bmatrix}
\end{align*}
\]

The system output is given by the COD concentration, i.e., \( y = h(x) = x_4 \). By computing the Lie derivative of the system output \( h(x) \) along the vectors fields \( f(x) \) and \( g(x), \) it is obtained that \( L_f h(x) = (x_{x,\text{in}} - x_4) \). Since for normal operating conditions \( (x_{x,\text{in}} - x_4) > 0 \), it is easy to shown that the system (8) has a relative degree \( r = 1 \) which is well-defined out of the wash-out condition. In this way, such a system can be written in the canonical form (1) as follows:

\[
\begin{align*}
\dot{z} &= (x_{4,\text{in}} - z)D - k_1 \mu_1 v_3 (x_{4,\text{in}} - z)^a \\
\dot{v}_1 &= -k_1 \mu_1 v_1 (x_{4,\text{in}} - z)^a - v_2 \\
\dot{v}_2 &= \frac{v_2 v_3}{v_4} (x_{4,\text{in}} - z)^a [k_2 \mu_1 v_4 - k_1 \mu_2] \\
\dot{v}_3 &= \frac{v_3 v_5}{v_4} (x_{4,\text{in}} - z)^a [k_4 \mu_1 v_4 - k_5 \mu_2 - qe/v_5] \\
\dot{v}_4 &= v_4 (\mu_1 - \mu_2) \\
\dot{v}_5 &= \mu_4 v_5 [1 - \alpha k_1 v_2 (x_{4,\text{in}} - z)^{a-1}]
\end{align*}
\]

It has been proved that subsystem (10.2) is globally stable under normal operation conditions (Méndez-Acosta et al., 2002). In addition, the system output \( y = x_4 \) is available for feedback (Steyer et al., 2002b). Then, the anaerobic digestion model (8) satisfies the properties (P1)-(P3). In consequence, system (10) can be rewritten in the form (3) as follows:

\[
\begin{align*}
\dot{z} &= -k_1 \mu_1 v_3 (x_{4,\text{in}} - z)^a + (x_{4,\text{in}} - z)D \\
\dot{v}_1 &= \zeta(z, v, w)
\end{align*}
\]

where \( a = 0, \zeta(z, w) = \left( k_1 \right. \zeta(x_{4,\text{in}} - z)^a \right) \) \( w = x_{4,\text{in}} \) and \( (z, w) = (x_{4,\text{in}} - z)D \).

4.2 Anti-windup Scheme for Substrate Regulation

Once have been proved that model (8) satisfies the assumptions (A1)-(A3) and the properties (P1)-(P3), it is easy to shown that for the case (C1) the following control laws is obtained:

\[
\begin{align*}
\begin{bmatrix}
\dot{z} \\
\dot{\zeta}
\end{bmatrix} &= \begin{bmatrix}
\hat{\eta} + (x_{4,\text{in}} - z)D_{\text{sat}} + g_1(z - \hat{z}) \\
\hat{\eta} = g_1(z - \hat{z}) - \frac{1}{(x_{4,\text{in}} - z)} \left[ \hat{\eta} + Kp(z - x_4^*) \right]
\end{bmatrix}
\end{align*}
\]

where \( \hat{\eta}_{4,\text{in}} = 16.0 \, \text{g/L} \) is the nominal value of the perturbation for the vinasses feeding into the reactor used to validate model (8) (see Steyer et al., 2002a). Now, by considering the case (C2) and assuming without lost of generality that \( \hat{\eta}_{4,\text{in}} = 16.0 \, \text{g/L} \) a control law of the form (7) can be obtained. Such a control law is given by the following linear controller

\[
K(s) = \begin{bmatrix}
g_1 & 1 & g_1 & 0 & 1/\bar{x} \\
g_2 & 0 & g_2 & 0 & 0 \\
0 & -1/\bar{x} & Kp/\bar{x} & -Kp/\bar{x} & 0
\end{bmatrix}
\]

in conjunction with the saturated control input, \( D_{\text{sat}} \).
4.3 Numerical Simulations

In this subsection, the performance and robustness of the control laws (12) and (13) are evaluated via numerical simulations. In order to observe the performance of such control laws in the servo-control and regulation problems, four different set-point changes were made on the effluent COD concentration. The model parameters used in the simulations are the following: \( k_1 = 42.14, \ K_{H1} = 16.0, \ k_2 = 116.5, \ \mu = 0.5, \ k_3 = 268.0, \ \bar{z}_{max} = 0.05, \ k_4 = 50.6, \ z_{max} = 0.031, \ k_5 = 343.6, \ K_{S1} = 7.1, \ k_6 = 435.0, \ K_{S2} = 9.28, \ k_{la} = 0.825, \ K_{St} = 16.0 \) and \( P_T = 1.0434 \). While the states initial conditions used are given by: \( x_1(0) = 0.1358, \ x_2(0) = 0.4829, \ x_3(0) = 76.1, \ x_4(0) = 0.9890, \ x_5(0) = 8.559 \) and \( x_6(0) = 67.186 \). Such values of the plant parameters have been identified for a pilot scale digester (Bernard et al., 2001). The control parameters used along the simulation were the same for both control laws and these are given by: \( g_1 = 1.5, \ g_2 = 0.5 \) and \( K_p = 2.0 \), which were used by Méndez-Acosta et al., (2003) is unconstrained control.

Figure 3 shows the dynamical behavior of the system output, i.e., the COD effluent concentration, \( x_4 \), and the time-response of the controller. The dotted line corresponds to the nonlinear control with no anti-reset windup (which was reported by Méndez-Acosta et al., 2003) whereas the solid line shows the performance of controller (12). Note that performance degradation is diminished by the anti-windup scheme. In analogous manner, the Figure 4 shows the closed-loop performance. The solid line corresponds to the linear control law (which was reported by Méndez-Acosta et al., (2003) whereas the solid line shows the performance of the controller \( K(s) \) in Eq. (13). Here, the anti-windup scheme also avoids the degradation of the controller performance.

Figure 2. COD influent concentration, \( x_{in} \).

Figure 3. Performance of the nonlinear control (12) (solid line) and (4) (dotted line).

Figure 4. Performance of the controller from Eq. (13) (solid line) and Eq. (7) (dotted line).
5. CONCLUSIONS

An anti-reset windup scheme for a class of nonlinear systems has been proposed in this paper. The anti-windup strategy is based on an observer structure. The degradation of the controller performance due to saturation is diminished by the proposed strategy. An anaerobic digester was used as illustrative example. The dynamical model of the digester satisfies all properties of the class system and the assumptions made for the anti-windup design. Simulations from an identified model were carried out to illustrate the results. Experimental implementation is a pilot scale digester is expected. Results in this direction will be reported as soon as available.

REFERENCES


