Active Control of Combustion Instabilities Using Model-Based Controllers *

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Abstract
This paper presents an implementation of active control of thermoacoustic instabilities on a swirl-stabilized spray combustor. Loudspeakers were used as control actuators in the closed-loop control scheme. Pressure transducers located in the combustion chamber produced the feedback signal in the control study. Experimental models of the combustor dynamics were developed using a non-parametric identification method. LQG, LQG/LTR and $H_\infty$ loop-shaping controllers were derived and tested in simulation as well as experimentally. Phase-delay control was used as a baseline method to compare the performance of these different controllers. The advantages of the model-based controllers over the baseline strategy are clearly presented.

Nomenclature

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
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<tbody>
<tr>
<td>D</td>
<td>combustion chamber diameter</td>
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<tr>
<td>H(s)</td>
<td>transfer function</td>
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<tr>
<td>$\bar{H}(\omega)$</td>
<td>frequency response data</td>
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<tr>
<td>$|H(s)|_\infty$</td>
<td>infinity norm of H(s)</td>
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<tr>
<td>K</td>
<td>controller</td>
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<td>L</td>
<td>combustion chamber length</td>
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<tr>
<td>LQG</td>
<td>Linear Quadratic Gaussian</td>
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<tr>
<td>LTR</td>
<td>Loop Transfer Recovery</td>
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<tr>
<td>P</td>
<td>pressure measurement</td>
</tr>
<tr>
<td>$P_{\text{rms}}$</td>
<td>fluctuating pressure</td>
</tr>
<tr>
<td>$Q_{\text{primary}}$</td>
<td>secondary air flowrate</td>
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<tr>
<td>$Q_{\text{secondary}}$</td>
<td>single-input single-output system</td>
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<tr>
<td>$(Q, R, Q_n, R_n)$</td>
<td>auto-spectral density of u</td>
</tr>
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</table>

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Introduction

Thermoacoustic instabilities involve a feedback cycle between the acoustic properties and the heat release dynamics of the combustor resulting in strong pressure oscillations. These instabilities can lead to severe structural degradation of the combustor and unacceptable noise pollution. The complexity of the combustion process and the lack of suitable actuators make active control of combustion instabilities a challenging problem.

Active control of gaseous-fueled combustion has received considerable attention (see Annaswamy et al., 1995; Gutmark et al., 1993; McManus et al., 1991; Schadow et al., 1992). Both fuel-feed modulation and air-feed modulations have been employed to control the instabilities. In liquid fueled combustion, time delays associated with atomization, droplet vaporization, and various droplet interactions complicate the use of liquid fuel modulation as a control input; however, control has been successfully demonstrated in such systems (see Yu et al., 2000; Yu et al., 1997; Cohen et al., 1998; Murugappan et al. 1999). The situation is further complicated in swirling flows where asymmetric heat release fluctuations generated by swirl can in some instances excite transverse acoustic modes of the combustor as was shown by Paschereit and Polifke (1998a). Research by Paschereit et al. (1998a, 1999) on a conical premixed burner showed that either acoustic forcing of the air streams or fuel-feed modulation at the appropriate phase instances relative to the pressure oscillations are effective in controlling instabilities for swirl-stabilized combustors. Their work also showed the potential of using asymmetric fuel modulations to suppress azimuthal instability modes that are excited by the presence of swirl.

The simplest control strategy in combustion control, phase-delay, is motivated by the observations of Lord Rayleigh (1976). He noted that the phase relation between the acoustic waves and the heat released by the flame determines the amplification/damping of pressure oscillations. The attenuation factors obtained with phase-delay control are usually moderate and sensitive to changes in the operating conditions of the combustor. In addition, phase-delay control can sometimes excite other instability modes after closing the control loop. This has motivated the implementation of more elaborate control techniques such as model-based control. However, model-based controllers require a representation of the combustor dynamics. Although some simplifications in the dynamic equations can be applied, the resulting analytical models are still very complex. Research on model-based techniques for control of combustion instabilities began mainly in the 1990s. Annaswamy et al. (1997) have developed a model for a simple premixed laminar combustor in order to design LQG controllers. Also, Chu et al. (1998) proposed a nonlinear model derived for a bluff-body flame holder and $H_\infty$ loop-shaping control design was applied. The results obtained by simulation of the nonlinear model were quite successful. Similarly, Hong et al. (2000) presented the design of an $H_\infty$ robust controller for suppressing combustion instabilities in propulsion systems. Annaswamy et al. (2000) reported experimental results of $H_\infty$ controllers in a small combustor test-rig (1kW) with laminar flow. Recently, Campos et al. (2001) presented the development of experimental models to design $H_\infty$ controllers for a swirl stabilized premixed burner. On the other hand, adaptive control (see Billoud et al., 1992; Evesque and Dowling, 1998) has also been suggested as an alternative control strategy to suppress combustion instabilities. However, the convergence of the adaptive schemes is not always guaranteed. Analytical models of large-scale combustors are complex and difficult to derive. For this reason, a new approach in combustion instability modeling has also been explored by Paschereit et al. (1998b) and Schuermans et al. (2000), where the combustion system is modeled as a network of acoustic elements. Hence, the acoustics, flame and burner dynamics are modeled based on acoustic properties using experimental data.

This paper presents a comparison between phase-delay and experimental model-based con-
controllers that were implemented on a swirl-stabilized spray combustor. Loudspeakers were used as the actuators in the feedback control studies due to their more efficient and linear response. Due to the lack of suitable analytical models to describe the interactions inside a swirl stabilized combustor, models based on experimental measurements were developed. These experimental models were based on the acoustic properties of the system. Optimal control strategies such as LQG, LQG/LTR, and $H_\infty$ loop-shaping were applied in the controller design. The advantages of these controllers over the baseline phase-delay strategy are presented in this paper.

Experimental Test-Rig

Figure 1: Experimental Test-Rig
Active control tests were performed on a 125kW swirl-stabilized spray combustor. The combustor facility consists of two sections: the settling chamber and the combustion chamber. Two concentric pipes serve as the primary and secondary air settling chambers as shown in Figure 1. A honeycomb was placed in the primary and secondary chambers respectively to avoid large scale eddies in the air flow. Each settling chamber has an array of eight loudspeakers (Sanming 75A) that are mounted at equal polar angles about the chamber’s circumference. The primary and secondary air from the settling chambers are accelerated into the combustion chamber through concentric large area ratio nozzles. Swirl vanes having a 45° co-swirl orientation are placed at the exit of the nozzles. Uniformly located between the coaxial air jets are eight pressure-fed atomizers. Four of the eight fuel sprays are operated at a constant fuel-rate and the other four are modulated using automotive-type fuel injectors. The modulated fuel sprays deliver 1.8gph of ethanol fuel, while the remaining four provide a constant supply of fuel at 3.2gph.

![Combustor Block Diagram with Acoustic Control](image)

The circular combustion chamber has a diameter of 5.5in and a length of 22in. The exit of the combustion chamber was unrestricted. In addition, the chamber was water-cooled, although no measurements of the combustor wall temperature were taken during the experiments. High frequency response pressure transducers (Kistler 6061B and 7061B) were placed along the length of the combustion chamber wall to monitor the instability. A pressure sensor located at a distance of x/L=0.2 from the dump plane provided the feedback signal in the closed-loop control configuration. The pressure signal was sampled at a rate of 10 kHz and then processed using a dSPACE 1103 PPC Control Board. The control algorithms were implemented in discrete time using Matlab Simulink programs in conjunction with the dSPACE software. The control signals were then converted to continuous time by way of the DAC and sent to four 250 watt audio amplifiers (Radio Shack MPA-250) where the signals were amplified before being sent to the loudspeakers as shown in Figure 2. The audio amplifiers had a limitation of ±0.6V input. At all forcing levels however, there was no noticeable effect on the flame structure due to the acoustic forcing.
Operating Condition

The primary and secondary air flow rates were varied from 12 to 30 cfm and 30 to 100 cfm respectively to determine the behavior of the instability. The fuel flow rate was maintained at 5 gph throughout the testing. The combustor exhibited one dominant instability mode whose frequency changed with the air flow rates. This dominant mode varied linearly with the secondary air flow rate, roughly in the interval 250 to 300 Hz. The peak amplitude of the pressure oscillations corresponding to the instability frequency are shown in Figure 3 as a function of secondary air at four different primary airflow rate conditions. The air flow rates chosen in this study were 24 cfm for the primary air and 100 cfm for the secondary air as indicated by the marker in Figure 3. This particular flow condition was chosen since it presented a clear dominant instability frequency compared to the strongest instability condition \((Q_{\text{primary}} = 24 \text{ cfm}, Q_{\text{secondary}} = 50 \text{ cfm})\) that presented two instability modes. A sample time trace and power spectral density of the pressure fluctuations for this baseline condition are shown in Figure 4. The instability at the baseline condition occurred at a frequency of 250 Hz with the presence of a noticeable second harmonic at 500 Hz. A low-frequency component in the pressure fluctuations is also noted in Figure 4. The origin of this low-frequency mode could not be exactly determined. Nevertheless, from the observations made in the test-rig, it was clear that this was not a Lean-Blow-Out effect. The levels of oscillations within the combustor were of the order of 2% of the mean (atmospheric) pressure for the baseline condition.

\[
\begin{array}{|c|c|c|c|}
\hline
\text{Fuel-Flow Rate} & Q_{\text{primary}} & Q_{\text{secondary}} & \text{Pressure} \\
\hline
5.0 \text{gph} & 24 \text{ cfm} & 100 \text{ cfm} & \text{atmospheric} \\
\hline
\end{array}
\]

Table 1: Parameters of Baseline Condition

In the proposed study, linear model-based controllers were used for combustion control. These controllers process the input measurement and produce a proportional (continuous) signal to drive the actuators. Consequently, the actuators must be capable of taking this proportional signal, and have enough control authority to modify the combustion process. Loudspeakers have a linear response and can modify the flow field according with their driving voltage. Therefore, the feedback control was decided to be implemented using acoustic forcing. Since four of the fuel nozzles were configured with automotive fuel injectors, they are operated in an open-loop configuration modu-
lated at 400Hz relative to a dominant 250Hz instability; thus the baseline fuel modulations do not alter or affect the dominant instability. Acoustic forcing was used to identify the dynamics of the system from which experimental models were derived. Based on these models, feedback controllers were developed and implemented using the loudspeakers as control actuators.

**Instability Mechanism**

![Diagram](image)

Inside the combustor chamber, there is a complex interaction among different phenomena. The thermoacoustic instability is recognized as a feedback interaction between the flame (heat-release) and acoustic (pressure) characteristics. Based on our observations in the combustor, the instability dynamics follow the structure presented in Figure 5. Swirl is generated in the burner to improve mixing and to help to stabilize the flame. The flow turbulence and coherent structures induce random and periodic fluctuations respectively in the flow field. These produce variations in the equivalence ratio $\phi_m$ at the flame front that in return produce variations in the heat release $q$. The heat release oscillations are then amplified/attenuated through the acoustic dynamics that cause
pressure perturbations $P$. When a thermoacoustic instability is present, there is a feedback path in the system. This feedback path could be through a system-feed coupling or through periodic variations in the flame surface area (caused by vortex shedding, flapping, etc.). Consequently, the large pressure oscillations $P$ cause fluctuations in the equivalence ratio $\phi_e$ (see Peracchio and Proscia, 1998) and the feedback loop is closed. This feedback cycle originate a limit cycle oscillation (see Dowling, 1997) since the combustion dynamics show nonlinear characteristics that basically limit the growth of the periodic perturbations. In the acoustic control configuration, there are two control channels: primary and secondary. The loudspeakers then produce perturbations in the incoming air into the burner according with the input signals $u_p$ and $u_s$, which in turn produce perturbations in the equivalent ratio, $\phi_p$ and $\phi_s$. Therefore, the objective of the control scheme is to produce the proper driving signals to the loudspeakers to attenuate the instability in the system.

The blocks in Figure 5 could be roughly assumed to be linear except the blocks representing the combustion and mixing dynamics. The combustion dynamics are known to present a saturation-like nonlinear behavior (see Dowling 1997) since there is a maximum amount of heat that can be released by the fuel. On the other hand, the swirl mixing introduce complex nonlinear interactions in the flow field. However, a linear approximation can be deduced since it presents a dominant mode characteristics. Moreover, the swirl/flow dynamics block is a continuously excited block since the fuel/air mixing is always present as long as the combustor is running. The acoustics can be treated as a linear mapping and the coupling block is linked to the acoustic impedance of the combustion chamber. Note that when the combustor is running in the stable regime, the coupling block is not present. Finally, the control channel blocks contain intrinsically the dynamics of the settling chamber and acoustic impedance of the concentric nozzles connecting the combustion chamber.

**Acoustic Modeling**

![Figure 6: Combustor Model Structure (a) 2 control channels and (b) 1 control channel](image)

The strategy to obtain the combustor model is presented in this section. The ultimate objective for the combustor model is the controllers design, and specifically, the design of linear-time invariant controllers such as LQG, LQG/LTR and $H_\infty$ loop-shaping. So, the model mainly should present the dynamics of the control signals to pressure measurements in a linear fashion. An analytical model of the combustor is a difficult task since the flow is characterized by turbulence and three-dimensionality. As a result, the partial differential equations describing the flow characteristics
become highly nonlinear and difficult to simplify. Hence, it was decided to obtain a linear approximation of the combustor dynamics using experimental input/output data. Therefore, for a given operating condition, the model is only based on experimental measurements (pressure fluctuations).

The combustor is then modeled as a network of acoustic elements. Based on the type of acoustic forcing, two combustor models were derived as shown in Figure 6. The models explore separate or combined forcing of the combustor. In Figure 6, \( u_p \) and \( u_s \) represent the control signals of the primary and secondary control channels respectively, \( u_{ps} \) denotes the control signal applied to both channels simultaneously, \( w \) represents white noise and \( P \) the pressure measurement. Comparing to Figure 5, the blocks \( H_{primary} \), \( H_{secondary} \) and \( H_{primsec} \) represent the lumped interactions of the actuators and heat release-pressure feedback mechanism. On the other hand, \( H_{source} \) represents the flow unsteadiness and mixing lumped with the instability interaction, which all together are linked to the acoustic properties of the chamber.

In the no forcing (\( u_s = u_p = 0 \) and \( u_{ps} = 0 \) in Figure 6) case, the model proposed is based on the assumption that the flame behaves as a noise source (see Klein, 2000). Consequently, the combustor is represented as a stochastic system. Hence, for this condition the combustor dynamics are presented as a continuously excited source modeled as colored white noise. Due to this continuous excitation to the system, this block model was artificially assumed to be driven by a white noise source. Hence in order to obtain this model, the pressure signal was recorded with no control applied to the combustor. This pressure data was transformed to the frequency domain to obtain \( \hat{H}_{source}(\omega) \). The experimental frequency data is shown in Figure 7. From this plot, the instability frequency is located around 250Hz, so it was chosen to obtain a model only in the interval 0 to 1kHz. Next, the magnitude of the frequency data was fitted with a stable transfer function \( H_{source}(s) \). The phase information was discarded since this block is assumed to be driven by a white noise source. A 12th order transfer function was needed to accurately fit the data.

![Figure 7: Experimental Frequency Response, \( \hat{H}_{source}(\omega) \), of Combustor Dynamics](image)

The acoustic forcing of the primary and secondary airflows is carried out by a set of 8 loudspeakers in each channel. Now, the identification of the combustor response to the actuators excitation was pursued. In the system identification method, band-limited white noise (0 to 1kHz) was applied to the speakers and the corresponding pressure signal was recorded. To avoid clipping of the acoustic amplifiers, the level of the white noise was limited to ±0.6V. In addition, due to the continuous excitation coming from the flame interactions, the pressure signal must be correlated with the identification input signal to be able to extract the response of the combustor to this test signal. Thus, the experimental frequency response \( \hat{H}(\omega) \) was computed by

\[
\hat{H}(\omega) = \sqrt{\frac{S_{uu}(\omega)}{S_{uy}(\omega)}}
\]
where $S_{uu}(\omega)$ and $S_{uu}(\omega)$ denotes the cross-spectral density in the frequency domain of the input to output and input with itself respectively. The next step is to fit a rational transfer function $H(s)$ to this frequency data $\hat{H}(\omega)$. It is of particular interest to fit the frequency response data with a stable transfer function since this will simplify the control design and help to simulate accurately the combustor model. It is important to mention that this representation from control signal-to-pressure measurement contains information about the actuator response (speakers), heat-release and pressure interactions to the control excitations.

Two identification strategies were pursued. In the first one, see Figure 6(a), each control channel (primary and secondary) was identified separately. So, the experimental frequency response corresponding to the mapping of primary speakers to pressure measurement was computed as $\hat{H}_{\text{primary}}(\omega)$. Similarly, another mapping from the secondary control speakers to the pressure measurement was derived as $\hat{H}_{\text{secondary}}(\omega)$. The corresponding frequency responses for these two mappings $\hat{H}_{\text{primary}}(\omega)$ and $\hat{H}_{\text{secondary}}(\omega)$ are plotted in Figures 8 and 9. Note that from the phase plot of $\hat{H}_{\text{secondary}}(\omega)$, there was a large time-lag associated with the secondary forcing.

In the second approach, the same white noise signal was applied simultaneously to both primary and secondary speakers. Consequently, only one response $\hat{H}_{\text{primsec}}(\omega)$ from the control speakers to the pressure measurement was obtained. In Figure 10, the frequency response of $\hat{H}_{\text{primsec}}(\omega)$ is shown. Note that the time-lag associated with the secondary forcing was still present in the frequency response.

The next step in the modeling process was to fit a rational transfer function to the frequency response data for each case. All the data sets were SISO, thus the models were realized by a frequency response.

The resulting model for $H_{\text{primary}}(s)$ was an 11th order stable and proper transfer function with non-minimum-phase zeros.

$$
H_{\text{primary}}(s) = \frac{-0.65404(s + 1.549 \times 10^4)(s^2 + 511.7s + 1.507 \times 10^6)(s^2 - 189.2s + 2.195 \times 10^6)}{(s + 9964)(s^2 + 119.8s + 2.056 \times 10^6)(s^2 + 45.16s + 2.387 \times 10^6)}
$$

$$
\frac{(s^2 + 112.7s + 2.927 \times 10^6)(s^2 + 394.9s + 1.805 \times 10^7)(s^2 - 65.69s + 3.821 \times 10^7)}{(s^2 + 451.1s + 2.808 \times 10^6)(s^2 + 406.6s + 1.689 \times 10^7)(s^2 + 40.68s + 3.733 \times 10^7)}
$$

The fitting of $\hat{H}_{\text{secondary}}(\omega)$ was more complicated, mainly due to time-lag presented in the frequency response and sudden hump in the phase response around the peak frequency. So, the fitting accuracy had to be sacrificed somewhat to obtain a stable approximation. The final representation for $H_{\text{secondary}}(s)$ was a strictly proper transfer function with 15 stable poles and 9 non-minimum phase zeros.

Finally, the model for the combined speakers response $\hat{H}_{\text{primsec}}(\omega)$ was obtained. The resulting approximation for $H_{\text{primsec}}(s)$ was a 14th order stable and proper realization with non-minimum phase zeros.
Figure 8: Experimental Frequency Response of Primary Control Channel $\hat{H}_{primary}(\omega)$

Figure 9: Experimental Frequency Response of Secondary Control Channel $\hat{H}_{secondary}(\omega)$

Figure 10: Experimental Frequency Response of Control Speakers Channel $\hat{H}_{primsec}(\omega)$
A brief description of the design techniques applied in the closed-loop control schemes are presented in this section.

**LQG Control Technique**

Suppose that the plant model is given by

\[
\begin{align*}
\dot{x} &= Ax + Bu + \Gamma w \\
y &= Cx + Du + v
\end{align*}
\]

where \(w(t)\) and \(v(t)\) represent probabilistic knowledge about the plant disturbances. These disturbances are assumed zero mean Gaussian white noises with covariance matrices:

\[
\begin{align*}
\mathbb{E}\{w(t)w^T(t)\} &= Q_n \geq 0 \\
\mathbb{E}\{v(t)v^T(t)\} &= R_n \geq 0
\end{align*}
\]

and uncorrelated

\[
\mathbb{E}\{v(t)w^T(t + \tau)\} = 0
\]

for all \(t\) and \(\tau\). \(\mathbb{E}\{\cdot\}\) is the probabilistic expectation operator. The Linear/Quadratic Gaussian (LQG) control problem is to devise a control law that minimizes the cost function:

\[
J = \lim_{T_o \to -\infty} \mathbb{E} \left\{ \frac{1}{T_o} \int_{0}^{T_o} \left[ x^T Q x + u^T R u \right] dt \right\}
\]

Using the separation principle the LQG control problem can be decomposed into two subproblems:

\[
H_{\text{secondary}}(s) = \frac{-8.673246 \times 10^{18} (s + 2496)(s^2 - 0.1699s + 5.324 \times 10^5)}{(s + 870.3)(s^2 + 23.29s + 5.225 \times 10^5)(s^2 + 105.5s + 1.987 \times 10^6)}
\]

\[
(s^2 + 95.25s + 2.058 \times 10^6)(s^2 - 22.79s + 2.419 \times 10^6)(s^2 + 131.9s + 2.695 \times 10^6)
\]

\[
(s^2 + 82.48s + 2.223 \times 10^6)(s^2 + 40.58s + 2.436 \times 10^6)(s^2 + 654.8s + 2.626 \times 10^6)
\]

\[
(s^2 + 56.49s + 2.658 \times 10^6)(s^2 + 16.18s + 1.304 \times 10^7)
\]

\[
H_{\text{primary}}(s) = \frac{-0.54483(s^2 + 62.46s + 4.759 \times 10^5)(s^2 + 415.5s + 1.601 \times 10^6)}{(s^2 + 89.94s + 5.171 \times 10^5)(s^2 + 94.18s + 1.94 \times 10^6)}
\]

\[
(s^2 - 197.7s + 2.204 \times 10^6)(s^2 + 31.46s + 2.372 \times 10^6)(s^2 + 3.359s + 2.699 \times 10^6)
\]

\[
(s^2 + 46.62s + 2.307 \times 10^6)(s^2 + 19.42s + 2.398 \times 10^6)(s^2 + 32.76s + 2.522 \times 10^6)
\]

\[
(s^2 + 29.66s + 3.75 \times 10^6)(s^2 + 5.241s + 6.114 \times 10^6)
\]

\[
(s^2 + 35.55s + 3.79 \times 10^6)(s^2 + 6.569s + 6.112 \times 10^6)
\]

In Figure 11, 12 and 13 the realization for each model is compared with the experimental frequency response data. A complete description and justification of this non-parametric identification technique can be found in Ljung (1999).
Figure 11: Comparison between Fitted Transfer Function $H_{primary}(s)$ and Experimental Data $\tilde{H}_{primary}(\omega)$

Figure 12: Comparison between Fitted Transfer Function $H_{secondary}(s)$ and Experimental Data $\tilde{H}_{secondary}(\omega)$

Figure 13: Comparison between Fitted Transfer Function $H_{primsec}(s)$ and Experimental Data $\tilde{H}_{primsec}(\omega)$
• Application of the standard deterministic LQR (Linear/Quadratic Regulator) control problem

\[
\min_u \lim_{T_0 \to \infty} \int_0^{T_0} \left[ x^T Q x + u^T R u \right] dt
\]  

(10)

whose solution requires a state feedback control law.

• Obtaining of an optimal estimate \( \hat{x} \) of states \( x \) minimizing

\[
E \left\{ (x - \hat{x})^T (x - \hat{x}) \right\}
\]

(11)

using the Kalman filter theory.

Hence, the control design requires to specify the parameters \( Q \) and \( R \) related to the LQR problem, and \( Q_n \) and \( R_n \) parameters needed to compute the optimal Kalman estimator.

**LQG/LTR Design**

LQR controllers using state-feedback have certain guaranteed robustness properties. But, in real-life implementations it is not possible to have all states measurable. Instead, the LQG approach proposes to use a Kalman filter to provide state estimates for feedback so the feedback signal coming from the LQR design could be implemented. However, the robustness properties of the LQR design are not inherited by the LQG approach.

The LQG/Loop-Transfer Recovery (LQG/LTR) technique proposes to select the parameters of the LQG problem in such a way that the resulting closed-loop system recovers the robustness properties of the feedback system with LQR control. Thus, two parameters \( \rho \) and \( \mu \) are specified in design stage and reflect the trade-off between performance and robustness. For a more in depth explanation of the LQG and LQG/LTR design techniques refer to Doyle and Stein (1981).

**\( H_\infty \) Loop Shaping Technique**

The design technique incorporates the classical loop-shaping methods to obtain performance/robust stability tradeoffs, and a particular \( H_\infty \) optimization problem to guarantee closed-loop stability and a level of robustness at all frequencies. The design methodology uses only basic concepts of loop-shaping methods, commonly used in classical frequency based designs like lead-lag controllers, and a robust stabilization controller for a normalized coprime factor perturbed system is used to construct the final controller. Hence, the design strategy is based roughly on two steps. First, a compensator is designed to shape the open loop plant in the frequency domain according with the performance requirements. Finally, the controller is obtained through the minimization of an \( H_\infty \) problem using the shaped plant in order to provide robustness to the controller.

Design Procedure (SISO):

1. Loop-shaping : the frequency response of the open-loop plant is shaped using a compensator \( W(s) \) to give the desired open-loop shape. The nominal plant \( P(s) \) and the shaping compensator \( W \) are combined to form the shaped plant \( P_s \), where \( P_s = PW \). It is assumed that there is no pole-zero cancellation of unstable modes of \( P \).

2. Synthesize a stabilizing controller \( K_\infty(s) \) for \( P_s \), through solving

\[
1/b_{opt} = \min_K \left\| \begin{bmatrix} K \\ I \end{bmatrix} (I + P_s K)^{-1} \begin{bmatrix} I & P_s \end{bmatrix} \right\|_\infty
\]

(12)
3. Check the resulting parameter $b_{opt}$, if $b_{opt} \ll 1$ then return to (1) and adjust $W(s)$.

4. The final feedback controller $K(s)$ is then constructed by combining the $H_\infty$ controller $K_\infty$ and the shaping compensator $W$, such that

$$K = K_\infty W$$

5. If it is necessary apply model reduction to the resulting controller $K$ (Balance Truncation)

For a more detailed description of this design technique and justification for $H_\infty$ loop-shaping refer to Zhou and Doyle (1998).

**Controller Synthesis**

LQG, LQG/LTR and $H_\infty$ loop-shaping designs were applied to the combustor models $H_{primary}$, $H_{secondary}$ and $H_{primsec}$. From the description of the combustor model shown in Figure 6, the control formulation can be considered as an output disturbance attenuation problem (see Zhou and Doyle, 1998). It is well known that the optimal controller tends to invert the dynamics of the plant. However, the three models given in (2), (3) and (4) are non-minimum phase. Consequently, it is expected that the controllers derived from LQG, LQG/LTR and $H_\infty$ loop-shaping designs tend to be unstable since they perform some kind of approximated inversion in a limited frequency range. So, there is a tendency to have unstable controllers. Nevertheless, special care was taken to select the design parameters properly in order to obtain stable controllers. However, some performance degradation should be expected.

![LQG Controllers Frequency Response](image)

**Figure 14: LQG Controllers Frequency Response**

The LQG controllers were designed based on the following design parameters: $Q = 1 \times 10^3$, $R = 1$, $Q_n = 10$ and $R_n = 10$. These set of parameters produced stable controllers for each model. The frequency responses of the resulting controllers are plotted in Figure 14. For the LQG/LTR design, the parameters $\mu = 1$ and $\rho = 0.1$ guaranteed stable controller for each case. In Figure 15, the LQG/LTR controllers are plotted. In the $H_\infty$ loop-shaping design, a weighting function $W(s)$ shown in Figure 16 was used to stress the importance of the attenuation at the instability frequency:

$$W(s) = \frac{3.95 \times 10^6(s + 0.6283)}{(s + 314.2)(s^2 + 3016s + 6.317 \times 10^4)}$$
Figure 15: LQG/LTR Controllers Frequency Response

The bode plots of the resulting controllers are shown in Figure 17.

Figure 16: Weighting Function for $H_{\infty}$ loop-shaping Design

Note from the frequency responses of Figures 14, 15 and 17, the controllers are doing approximate inversions of the corresponding plants, roughly in the frequency range 200 to 300Hz, which are evident from the magnitude plots and the humps in the phase response. Now, denote $K_{\text{primary}}$, $K_{\text{secondary}}$ and $K_{\text{primsec}}$ the controllers designed using the models $H_{\text{primary}}$, $H_{\text{secondary}}$ and $H_{\text{primsec}}$ respectively. The closed-loop transfer functions (cf. Figure 6) from $w$ to $P$ are given by

$$T_{\text{pLCS}} = \frac{H_{\text{source}}}{1 + H_{\text{primary}}K_{\text{primary}} + H_{\text{secondary}}K_{\text{secondary}}}$$

$$T_{\text{primsec}} = \frac{H_{\text{source}}}{1 + H_{\text{primsec}}K_{\text{primsec}}}$$

for separate and combined primary and secondary forcings respectively. In Figure 18, the frequency responses for these closed-loop transfer functions are shown for the LQG controllers. Now, the
On the other hand, in order to facilitate the controller implementation, the order of the controllers was limited to be 10th order for all the strategies. For this purpose, balance truncation (see Zhou and Doyle, 1998) was applied such that the shape of the frequency response was not significantly altered. In addition, the continuous-time controllers were transformed to discrete by ‘bilinear transformation’ with ‘pre-warping’ at 250Hz and a sampling frequency of 10kHz, and they were implemented by the dSPACE data-acquisition system.

**Acoustic Control Implementation**

The implementation of the linear controllers earlier designed is shown in this section. First, simulation was computed to judge the performance of the controllers and finally the experimental implementation was carried out. Phase-delay was used as the baseline strategy to compare the
controllers response.

**Performance Indexes**

Two performance indexes were chosen to judge the controllers performance. First, the peak attenuation (PA) is defined as

$$PA = 10 \cdot \log_{10} \left[ \frac{\max_{\omega} P_{\text{control}}(\omega)}{\max_{\omega} P_{\text{open-loop}}(\omega)} \right] \text{dB}$$

(13)

where $P(\omega)$ is the discrete Fourier transform of the pressure time trace $P(t)$. An estimate of the power of the pressure fluctuations was obtained by

$$Pressure \ Power = \frac{1}{N} \sum_{\omega} |P(\omega)|^2$$

where $N$ is the number of samples in the pressure trace. The reduction in the overall pressure power (NRR) was then defined as

$$NRR = 10 \cdot \log_{10} \left[ \frac{Pressure \ power_{\text{control}}}{Pressure \ power_{\text{open-loop}}} \right] \text{dB}$$

(14)

**Closed-Loop Simulation**

In this section, the simulation of the closed-loop control scheme with the combustor model in Figure 6 was pursued. The simulation was carried out for cases (a) and (b) in Figure 6: separate and combined control channels. The corresponding simulation blocks in Figures 20 and 21 were used. Note that $K_{\text{primary}}$, $K_{\text{secondary}}$ and $K_{\text{primsec}}$ represent the controllers for the primary, secondary and combined channels respectively.

In the experimental implementation of the phase-delay control, a band-pass filter with cut-off frequencies of 80Hz and 500Hz was used to reject noise coming from low and high frequency components. The computation of the band-pass filter $H_{\text{bandpass}}(s)$ was separated into a low-pass butterworth filter with cut-off frequency of 500Hz and a high-pass butterworth filter with cut-off frequency of 80Hz. Since these two cut-off frequencies are close to each other, there is some
Openloop Combustor Dynamics

Primary Forcing Acoustic Dynamics

Secondary Forcing Acoustic Dynamics

Primary Controller

Secondary Controller

primary control signal

secondary control signal

white noise

gain

pressure measurement

time to start control

tstart

t

Figure 20: Simulation Block with Separate Control Channels

Openloop Combustor Dynamics

Primary and Secondary Forcing Acoustic Dynamics

Primary and Secondary Controller

combined control signal

sum

primary control signal

primary and secondary control signal

gain

gain

pressure measurement

time to start control

tstart

t

Figure 21: Simulation Block with Combined Control Channel
phase shift at the instability frequency \( \approx 250\text{Hz} \) coming from the filtering process. In order to validate the accuracy of the experimental combustor model shown in Figure 6, simulation with the same band-pass filter was performed to determine the effect of phase-delay control in the modeled pressure measurement. The simulation as well as the experimental testing was carried out by choosing independent delay and gain factors for primary, \( k_p \) and \( \tau \), and secondary, \( k_s \) and \( \tau_s \), control channels. Consequently, the controllers have the form:

\[
K_{primary} = k_p e^{-\tau_p s} H_{bandpass}(s) \\
K_{secondary} = k_s e^{-\tau_s s} H_{bandpass}(s)
\]

The relative time-delays between the pressure oscillations and the primary and secondary air forcing were varied to find the optimum delays for suppressing the instability. Figure 22(a) and (b) show the surface plots for the attenuation/amplification of the peak frequency and pressure power evaluated in the interval 0-1kHz as functions of the forcing delays. The values in the plots are normalized by the maximum dB amplification. Thus, positive values represent amplification with a maximum value of 1 and negative values are indicative of attenuation of the instability. Control of the instability was seen to be less sensitive to secondary air forcing, and the majority of phases resulted in an amplification of the instability. The maximum reduction was found at a primary and secondary phase delay of 0° relative to the instability.

![Figure 22](image)

Figure 22: Simulation Surface for Phase-Delay Control: (a) Peak Frequency Magnitude and (b) Power Amplification/Reduction as a Function of Primary and Secondary Forcing Delays

Now, the simulation of the linear controllers LQG, LQG/LTR and \( H_\infty \) loop-shaping was pursued. The simulation showed that the best control scheme for the LQG design was a combined forcing. Meanwhile, for the LQG/LTR and \( H_\infty \) loop-shaping a separate control produced the best attenuation. Table 2 presents a summary of the simulation control performance.

<table>
<thead>
<tr>
<th></th>
<th>Phase-Delay (dB)</th>
<th>LQG (dB)</th>
<th>LQG/LTR (dB)</th>
<th>( H_\infty ) loop-shaping (dB)</th>
</tr>
</thead>
<tbody>
<tr>
<td>NRR</td>
<td>-5</td>
<td>-5</td>
<td>-8</td>
<td>-7</td>
</tr>
<tr>
<td>PA</td>
<td>-6</td>
<td>-7</td>
<td>-8</td>
<td>-7</td>
</tr>
</tbody>
</table>

Table 2: Summary of Controllers Simulation Performance
Experimental Implementation

The control strategies were now implemented in the test-rig. In the open-loop condition, no acoustic forcing is presented and the secondary fuel is being modulated in an open-loop fashion at a fixed frequency of 400Hz. The fuel modulation remained constant at this condition during all tests and was not used to control the instability. Moreover, since the instability frequency is $\approx 250$Hz, the modulated fuel did not excited or attenuated the instability. First, the phase-delay control was tested. As in the simulation case, independent delay and gain factors for primary and secondary control channels were selected to achieve maximum pressure suppression. Figure 23(a) and (b) show the surface plots of the attenuation/amplification of the pressure oscillations power and peak frequency values. The experimental results in Figure 23 are in good agreement with the simulations shown in Figure 22. Thus, the simulation of the experimental combustor model showed a great success in predicting the qualitative trend for phase-delay control, see Figure 22. In both the simulations and experiments, the maximum reduction of the instability occurred at a primary and secondary phase delay of $0^\circ$ relative to the instability. Thus, by coincidence, the phase shift provided by the band-pass filter was optimum. Figure 24 presents a plot of the experimental pressure spectra and time traces for the open-loop and optimal delay conditions. Figure 24 shows that phase-delay control reduced both the peak and power of the pressure oscillations by approximately 3dB.

In addition to the phase-delay control tests, controllers based on the LQG, LQG/LTR and $H_\infty$ loop-shaping control techniques were also tested. As mentioned earlier in the paper, system identification procedures were used to develop experimental models of the combustor for the two control configurations: separate and combined channels. For each of the three models $H_{primary}$, $H_{secondary}$ and $H_{primsec}$, controllers were designed using the techniques outlined earlier. During the experimental testing, delays for the primary and secondary control signals were adjusted to obtain maximum attenuation of the pressure oscillations and compensate for any uncertainty coming from the modeling.

Experiments showed that the best LQG design was obtained from a combined primary/secondary model. Note that the same behavior was observed in the simulation. Figure 25 shows the pressure spectra and time trace obtained from experiments when the LQG controller was applied. Both the fundamental (250Hz) and the second harmonic were reduced by about 10dB and 14dB respectively in the experimental response. The total power of the pressure oscillations was also reduced by 7dB.
This was substantially more than the phase-delay control, which at best resulted in 3dB peak and power attenuation.

The best attenuation with the LQG/LTR control was obtained when separate controllers for the primary and secondary forcings were implemented. This behavior was also seen in the simulations. Figure 26 gives the experimental pressure spectra and time traces for the best LQG/LTR control. The peak of the instability was attenuated by approximately 12dB, while the power of the pressure oscillations was reduced by 9dB. The figure also shows that the LQG/LTR control resulted in the energy of the fundamental instability mode being divided into 3 different frequencies centered about the open-loop instability peak frequency. The simulation also showed a division of the fundamental frequency into three modes.

The best $H_\infty$ loop-shaping controller was obtained with separate controllers for the primary and secondary air forcing. As in the previous two cases, simulation successfully predicted this behavior. The experiments showed 11 dB peak attenuation and 8dB power attenuation. The pressure data for this control case is shown in Figure 27. Note that applying control resulted in the peak instability being distributed between multiple frequencies all centered about the original instability frequency.

The overall experimental results are summarized in Table 3. Both the attenuation in the peak frequency magnitude and the power are shown. As seen in the individual plots, the LQG, LQG/LTR and $H_\infty$ loop-shaping result in greater reductions of the pressure oscillations than phase-delay.

<table>
<thead>
<tr>
<th></th>
<th>Phase-Delay (dB)</th>
<th>LQG (dB)</th>
<th>LQG/LTR (dB)</th>
<th>$H_\infty$ loop-shaping (dB)</th>
</tr>
</thead>
<tbody>
<tr>
<td>NRR</td>
<td>-3</td>
<td>-7</td>
<td>-9</td>
<td>-8</td>
</tr>
<tr>
<td>PA</td>
<td>-3</td>
<td>-10</td>
<td>-12</td>
<td>-11</td>
</tr>
</tbody>
</table>

Table 3: Summary of Controllers Experimental Performance

**Conclusions**

An experimental combustor model based on acoustic properties of the system was proposed. This identified model was able to provide the basic characteristics of the combustion process essential
Figure 25: Experimental Pressure Spectra and Time Traces for Open-loop and LQG Control

Figure 26: Experimental Pressure Spectra and Time Traces for Open-loop and LQG/LTR Control

Figure 27: Experimental Pressure Spectra and Time Traces for Open-loop and $H_{\infty}$ loop-shaping Control
for the design of model-based controllers. Even though the models were just an approximation of the actual system in a limited frequency range, they were able to provide sufficient information about the combustor to attenuate the pressure pulsations significantly. The closed-loop simulation predicted the same qualitative trends observed in the experimental testing. Compared to phase-delay, the model-based controllers resulted in better attenuation of the pressure oscillations. These results signify that more effective control can be obtained when the controller has knowledge of the dynamics of the system.

References


