Advanced Optimization - Assignment 11

1. Solve the following problem using (1) gradient projection for bounded QP's, and (2) a quadratic penalty method. In both cases, discuss briefly the details of your algorithm, the parameters used, and the number of iterations required for convergence.

   (a) $\min y(x) = -1 - 2x_1 - x_2 + x_1x_2 + x_1^2$,
   subject to $0 \leq x_1 \leq 3$ and $1 \leq x_2 \leq 4$.

2. Solve the following problems using an exact penalty method that does not rely on derivative information. In each case, discuss briefly the details of your algorithm, the parameters used, and the number of iterations required for convergence.

   (a) $\min_{x \in \mathbb{R}^2} f(x) = -2x_1 + x_2$, subject to

   $$\begin{aligned} (1 - x_1)^3 - x_2 &\geq 0, \\ x_2 + 0.25x_1^2 - 1 &\geq 0. \end{aligned}$$

   (b) $\min \sin(x_1 + x_2) + x_3^2 + \frac{1}{3}\left(x_4 + x_5^4 + \frac{1}{2}x_6\right)$ subject to

   $$\begin{aligned} 8x_1 - 6x_2 + x_3 + 9x_4 + 4x_5 &= 6 \\ 3x_1 + 2x_2 - x_4 + 6x_5 + 4x_6 &= -4. \end{aligned}$$

   (c) $\min 2x_1x_2 + 2x_1x_3 + x_2x_3$,
   subject to $x_1x_2x_3 = V^*$ and $x_i \geq x_{\min}$, $i = 1, 2, 3$, with $V^* = 20$ and $x_{\min} = 1.9$.