Advanced Optimization - Assignment 2

1. The following univariate example shows that strict local solution are not necessarily isolated. Consider

$$\min_x x^2 \quad \text{subject to } c(x) = 0,$$

   where

$$c(x) = \begin{cases} x^6 \sin(1/x) & \text{if } x \neq 0, \\ 0 & \text{if } x = 0. \end{cases}$$

   (a) Show that the constraint function is twice continuously differentiable at all x (including $x = 0$) and that the feasible points are $x = 0$ and $x = 1/(k\pi)$ for all nonzero integers k.

   (b) Verify that each feasible point except $x = 0$ is an isolated local solution by showing there is a neighborhood $\mathcal{N}$ around each such point within which it is the only feasible point.

   (c) Verify that $x = 0$ is a global solution and a strict local solution, but not an isolated solution.

2. Is an isolated local solution necessarily a strict local solution? Explain.

3. Solve the following problem graphically:

$$\min (x_1 - 3)^2 + (x_2 - 2)^2$$

   subject to

$$\begin{aligned} x_1^2 - x_2 - 3 &\leq 0, \\ x_2 - 1 &\leq 0, \\ x_1 \geq 0. \end{aligned}$$

4. Suppose you want to design a desk with length x and width y and the following properties:

   - The surface xy must be as large as possible.
   - The perimeter must not exceed 8 units.
   - The desk should not be too wide; that is, $y \leq b$.

   Solve this problem graphically for $b = 1$. What happens to the optimal surface when b increases?