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Three-dimensional model of SL(2,R) and visualization of SL(2,Z) as a pattern on the cubic lattice

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It is known that real special linear group $SL(2,\mathbb{R})$ is embedded into the three-dimensional sphere [1]. We can see the three-dimensional sphere by the stereographic projection. Through this visualization, every matrix in $SL(2,\mathbb{R})$ is realized as a point in the three-dimensional Euclidean space $\mathbb{R}^3$. In this talk, we propose another three-dimensional model of $SL(2,\mathbb{R})$. With this model, we can visualize $SL(2,\mathbb{Z})$ as a pattern of points on cubic lattice in $\mathbb{R}^3$. For the purpose of this visualization, we combine two software: Python as CAS, and GeoGebra as DGS. In this model, the set of matrices with the fixed value of trace forms a quadratic surface (hyperboloid of two sheets, double cone, or hyperboloid of one sheet) depending on the value of trace. Hyperbolic paraboloid also comes out as the surface of the fixed value of element. With these familiar surfaces, we can analyze the pattern of $SL(2,\mathbb{Z})$.

Keywords
Three-dimensional model, $SL(2,\mathbb{R})$, $SL(2,\mathbb{Z})$, Quadratic surface

References
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