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Animation of some mechanical systems with Mathematica

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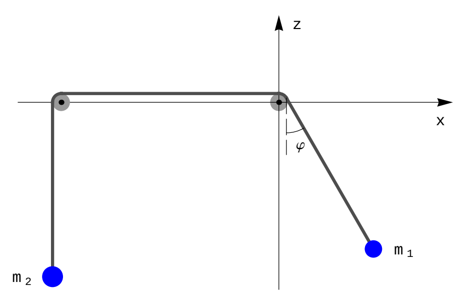
It is well known that the computer algebra system Mathematica (see [1]) is a powerful tool for doing both numerical and symbolic computation. Its built-in functions $DSolve$ and $NDSolve$ enable to solve easily differential equations describing motion of different mechanical systems and to visualize the results. Besides, using these solutions, one can animate the system and demonstrate its motion what is very interesting and useful for education.

As an example, let us consider a generalized version of the simple Atwood machine (see [2]) when two bodies of masses m_1, m_2 ($m_2 \geq m_1$) are attached to opposite ends of a massless inextensible thread wound round two massless frictionless pulleys of negligibly small radius.



Two separated pulleys are used here to avoid collisions of the bodies. Body m_2 is constrained to move only along a vertical while body m_1 swings in a vertical plane. Such a system has two degrees of freedom and its motion is described by the following differential equations (see [3])

$$(1+\mu)\ddot{r} = r\dot{\theta}^2 - g(\mu - \cos\varphi),$$
$$r\ddot{\varphi} = -2\dot{r}\dot{\varphi} - g\sin\varphi. \qquad (1)$$

Here r is a length of the thread between pulley and body m_1, the angle φ describes deviation of the thread from the vertical, g is a gravitational constant, and parameter $\mu = m_2/m_1$.

Note that equations (1) are nonlinear and their general solution cannot be obtained in symbolic form. Numerical analysis has shown (see [3]) that even small oscillation of the body m_1 can modify the system motion significantly and some unexpected kind of motion such as quasi-periodic one can arise.

In the present talk we use a numerical solution of equations (1) obtained for some realistic values of the system parameters and discuss the problem of animation of the generalized Atwood machine with Wolfram Mathematica. Our aim is to describe step by step a process of constructing a graphical object used for animation and to demonstrate a final result.

The animations in PDF or HTML format can be produced by KeTCindy which the second author has developed.

Keywords
Atwood's machine, Simulation, Quasi-periodic motion, Mathematica

References

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